

Technical Appendix

“The technology and cost structure of a natural gas pipeline: Insights for costs and rate-of-return regulation”

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This technical appendix is organized as follows. Section 1 summarizes the assumptions and introduces the notation. Section 2 reviews the standard cases of a monopoly and a social planner. Section 3 examines the case of rate-of-return regulation and gives a concise presentation of Klevorick (1971) and Callen et al. (1976) who were the first to analytically examine the economics of rate-of-return regulation for a Cobb-Douglas technology. Section 4 details the ratios presented in the paper.

1. Assumptions and notations

Technology

We consider the simple point-to-point pipeline infrastructure studied in our paper and assume the Cobb-Douglas production function: $Q^\beta = K^\alpha E^{1-\alpha}$, where $\alpha = 8/11$ is the capital exponent parameter and $\beta = 9/11$ is the scale coefficient.

From that production function, one can define $E(Q, K) = {}^{1-\alpha}\sqrt{K^{-\alpha} Q^\beta}$ the variable input requirements function that gives the amount of energy needed to transport the output Q on a pipeline infrastructure that has a given fixed amount of capital input K . We let $E_Q(Q, K)$ (respectively, $E_K(Q, K)$) denote the derivative of the input requirement function with respect to the output (respectively, the capital) variable. With our technology parameters, $E_Q(Q, K) > 0$ and $E_K(Q, K) < 0$.

Input prices

We let e denote the market price of the energy input and r denote the market cost of capital faced by the firm.

Cost function

Following the argumentation presented in Appendix A of the paper, the long-run cost-minimizing amount of capital stock needed to transport the flow Q is:

$$K(Q) = \left(\frac{e\alpha}{r(1-\alpha)} \right)^{1-\alpha} Q^\beta, \quad (1)$$

The long-run total cost function is $C(Q) = rK(Q) + eE(Q, K(Q))$ and thus:

$$C(Q) = \frac{r^\alpha e^{1-\alpha}}{\alpha^\alpha (1-\alpha)^{1-\alpha}} Q^\beta. \quad (2)$$

Demand

The inverse demand function is: $P(Q) = A Q^{-\varepsilon}$, where A is a constant and $1/\varepsilon$ is the absolute value of the price elasticity of demand. Here, it is assumed that: $\varepsilon < 1$ so that the total revenue obtained by a monopolist producing zero output is zero and that $\varepsilon > 1 - \beta$ so that the demand schedule always intersects the marginal cost schedule from above.¹

For notational convenience, we follow Callen et al. (1976) and introduce three parameters: (i) $\gamma \equiv \beta + \varepsilon - 1$, (ii) $\delta \equiv e\beta / [A(1-\varepsilon)(1-\alpha)]$, and (iii) $\eta \equiv \beta - (1-\varepsilon)(1-\alpha)$.

2. The cases of a monopoly and of a social planner

This section briefly reviews the standard outcomes obtained under two polar cases: (i) the profit-maximizing unregulated monopoly that charges a non-discriminatory price; and (ii) the hypothetical case of a welfare-maximizing social planner that behaves so as to maximize the sum of the producers' and consumers' surpluses (i.e., the net social welfare) while ensuring that the firm obtains zero economic profit. The latter case mimics the situation studied in Boiteux (1956).²

These two cases can be modeled using the optimization problems presented in Table TA-1. For concision, we omit the straightforward derivations of the first-order conditions and simply report the optimal decisions.

Note that in both cases: (i) the optimal amount of capital stock equals the cost-minimizing amount, that is, $K_M = K(Q_M)$ and $K_a = K(Q_a)$; and (ii) production is cost efficient as the equations $C(Q_M) = rK_M + eE(Q_M, K_M)$ and $C(Q_a) = rK_a + eE(Q_a, K_a)$ hold. Note also that, for the social planner, substitution of the optimal decisions Q_a and K_a in the zero profit condition (5) gives $P(Q_a)Q_a - C(Q_a) = 0$ which means that the output is set at a level such that the price equals the long-run average cost.

¹ These restrictions together impose that $1/\varepsilon$ is in the range (1,5.5) which is not a concern in our application.

² For concision, we omit the first-best solution that consists of solely maximizing the sum of the producers' and consumers' surpluses without paying attention to the firm's profitability. As this first-best solution entails establishing an output level for which price equals the long-run marginal cost, it compels the pipeline operator to operate at a loss, which is not realistic.

Table TA-1. The optimal decisions taken by a profit-maximizing unregulated monopoly and a welfare-maximizing social planner providing zero profit to the firm

	The unregulated monopoly	The welfare-maximizing planner that provides zero-profits to the firm
Optimization program	$\text{Max}_{K,Q} \Pi_M(Q) = P(Q)Q - rK - eE(Q, K) \quad (3)$	$\text{Max}_{K,Q} W(Q) = \int_0^Q P(q) dq - rK - eE(Q, K) \quad (4)$ s.t. $P(Q)Q - rK - eE(Q, K) = 0 \quad (5)$
Solution:		
Output	$Q_M = \left[\frac{A(1-\varepsilon)}{\beta} \left(\frac{\alpha}{r} \right)^\alpha \left(\frac{1-\alpha}{e} \right)^{1-\alpha} \right]^{\frac{1}{\gamma}} \quad (6)$	$Q_a = \left[A \left(\frac{\alpha}{r} \right)^\alpha \left(\frac{1-\alpha}{e} \right)^{1-\alpha} \right]^{\frac{1}{\gamma}} \quad (7)$
Capital	$K_M = \left(\frac{e\alpha}{r(1-\alpha)} \right)^{1-\alpha} (Q_M)^\beta \quad (8)$	$K_a = \left(\frac{e\alpha}{r(1-\alpha)} \right)^{1-\alpha} (Q_a)^\beta \quad (9)$

Note: The objective function (3) is the firm's profit, i.e.: the difference between the total revenue $P(Q)Q$ and the sum of the capital cost rK and the energy cost $eE(Q, K)$. The objective function (4) is the net social welfare defined as the sum of the consumer surplus $\int_0^Q P(q) dq - P(Q)Q$ and the producer's surplus $P(Q)Q - rK - eE(Q, K)$. The constraint (5) states that the firm is compelled to obtain zero economic profit.

Callen et al. (1976) define s_M the monopolist's rate of return on invested capital which is the ratio of: the accounting profit derived from the production of the output Q_M (that is: $P(Q_M)Q_M - eE(Q_M, K_M)$), and K_M the profit-maximizing capital stock: $s_M \equiv [\beta/(1-\varepsilon) - (1-\alpha)]r/\alpha$.

3. Rate-of-return regulation

We now assume that the infrastructure is provided by a private monopoly that is subject to rate-of-return regulation. This section briefly presents the theoretical literature on rate-of-return regulation for the special case of a Cobb-Douglas technology (Klevorick, 1971; Callen et al., 1976). It first reviews the behavior of the regulated monopoly before discussing the identification of a socially desirable rate of return.

3.1 The behavior of the regulated monopoly

The regulated monopoly is allowed to earn a fixed and exogenously-determined rate of return s that is lower than the rate of return s_M obtained by an unregulated monopolist (i.e., $s < s_M$).

The rate-of-return constraint stipulates that the monopoly's accounting profit (i.e., the total revenue $P(Q)Q$ minus $eE(Q, K)$ the cost of the variable input) cannot exceed the allowed return on invested capital sK . As the condition $s < s_M$ holds, the rate-of-return constraint is binding:

$$P(Q)Q - eE(Q, K) = sK, \quad (10)$$

The regulated firm is allowed to choose any combination of inputs (K and E) and output (Q) that jointly verifies the production function equation, and the rate-of-return constraint. Assuming profit maximization, the behavior of the regulated monopoly is thus determined by the following program:

$$\text{Max}_{K,Q} \quad \Pi(Q) = P(Q)Q - rK - eE(Q,K) \quad (11)$$

$$\text{s.t.} \quad P(Q)Q - eE(Q,K) = sK$$

$$K \geq 0, \quad Q \geq 0.$$

If the allowed rate of return is lower than the market cost of capital (i.e., $s < r$), profit maximization involves a corner solution: the firm's optimal decision is to withdraw from the market.

One must thus concentrate on the situation $s \geq r$. As shown in Klevorick (1971), the firm's optimal decisions must jointly verify the rate-of-return constraint (10) and the condition:

$$(s-r) \left[P'(Q)Q + P(Q) - eE_Q(Q,K) \right] = 0, \quad (12)$$

One can first examine the case $s > r$ where the allowed rate of return is larger than the market price of capital. The condition (12) indicates that the marginal revenue $P'(Q)Q + P(Q)$ must equal the regulated marginal cost $eE_Q(Q,K)$ which is the marginal cost of producing an additional unit of output when K is set at the level required to satisfy the rate-of-return constraint (10). Using that condition and the rate-of-return constraint (10), Callen et al., (1976) obtain the optimal decisions (K_R, Q_R) for a Cobb-Douglas production function and then evaluate: C_R the cost incurred by the regulated operator and W_R the net social welfare. Their results are summarized in Table TA-2.

Table TA-2. The optimal decisions taken by a regulated monopoly (case $s > r$)

Output	$Q_R = \left[\frac{A\delta - e}{s\delta^{1/\alpha}} \right]^{\alpha/\gamma}$	(13)
Capital	$K_R = \delta^{(1-\alpha)/\alpha} Q_R^{\eta/\alpha}$	(14)
Cost	$C_R = r\delta^{(1-\alpha)/\alpha} Q_R^{\eta/\alpha} + \frac{e}{\delta} Q_R^{1-\varepsilon}$	(15)
Net social welfare	$W_R = \frac{1}{1-\varepsilon} P(Q_R)Q_R - C_R$	(16)

In the specific case $s = r$, the allowed rate of return equals the market price of capital and the regulated firm is constrained to make at most zero economic profit. Klevorick (1971) highlights that the behavior of the regulated monopoly is indeterminate: the three combinations $(0,0)$, (K_a, Q_a) , and

(K_R, Q_R) evaluated with $s = r$ yield zero economic profit. To avoid that indeterminacy, we assume hereafter that the rate effectively implemented by the regulatory authority will be no less than r plus an infinitesimally small and positive increment. This rule imposes the choice of the combination (K_R, Q_R) .

3.2 The socially desirable rate of return

Klevorick (1971) and Callen et al. (1976) both examine the determination by a regulator of the fair rate of return s that maximizes the net social welfare given the regulated firm's reactions to that rate. They consider the two-level optimization problem:

$$\begin{aligned} \text{Max}_s \quad & W(s) = \int_0^Q P(q) dq - rK - eE(Q, K) & (17) \\ \text{s.t.} \quad & \left[\begin{array}{l} \text{Max}_{K, Q} \quad \Pi(Q) = P(Q)Q - rK - eE(Q, K) \\ \text{s.t.} \quad P(Q)Q - eE(Q, K) = sK \\ K \geq 0, Q \geq 0. \end{array} \right. \end{aligned}$$

We let s_R denote the solution to that program. The discussion above has shown that for a given rate of return s with $s_M > s > r$, the unique solution to the lower-level problem is the pair (K_R, Q_R) defined in Table TA-2 which is parameterized by s . Callen et al. (1976) thus reformulate the problem as a single-variable optimization problem:³

$$\text{Max}_s \quad W(s) = \int_0^{Q_R(s)} P(q) dq - rK_R(s) - eE(Q_R(s), K_R(s)). \quad (18)$$

The first-order condition for optimality yields the optimum value of the allowable rate of return s_R :

$$s_R = \frac{\eta^2 r}{\alpha [\beta - (1 - \alpha)(1 - \varepsilon)^2]}. \quad (19)$$

Note that, by assumption, the condition $0 < \varepsilon < 1$ holds, so the socially desirable rate of return s_R is lower than s_M the one obtained by the unregulated monopolist.

The rate s_R in (19) is valid if and only if, it verifies $s_R > r$, that is, if the elasticity and technological parameters are such that $\eta^2 > \alpha [\beta - (1 - \alpha)(1 - \varepsilon)^2]$. If that is not the case, the authority's best decision is to set s_R equal to r (plus an infinitesimally small and positive increment).

³ Note that this reformulation is rendered possible by their derivation of an analytical solution of the lower-level problem for the specific case of a Cobb-Douglas specification for the production function.

4. Static comparisons

To assess the performance of rate-of-return regulation, Callen et al. (1976) propose a series of ratios that are detailed in Table TA-3. These ratios are entirely determined by the ratio s/r , the demand elasticity and the technology parameters.

These ratios respectively compare:

- the output levels decided by: a private monopoly Q_M , a social planner applying the average-cost-pricing rule Q_a and a regulated monopoly Q_R ;
- the cost C_R incurred by the regulated firm and $C(Q_R)$ the cost that would have been incurred by a cost-minimizing firm producing the same output Q_R to assess the magnitude of the cost-increases caused by the Averch-Johnson effect (Averch and Johnson, 1962).
- the gain in net social welfare resulting from the regulation of a private monopoly ($W_R - W_M$) versus $(W_a - W_M)$ the gain in net social welfare resulting from the implementation of a social planner applying the average-cost-pricing rule in a monopolistically-controlled industry.

Table TA-3. The performance ratios

Output	$\frac{Q_R}{Q_a} = \left(\frac{r}{s} \frac{\eta}{(1-\varepsilon)\alpha} \right)^{\alpha/\gamma} \left(\frac{(1-\varepsilon)}{\beta} \right)^{\frac{1}{\gamma}}$
Output	$\frac{Q_R}{Q_M} = \left(\frac{r}{s} \frac{\eta}{(1-\varepsilon)\alpha} \right)^{\alpha/\gamma}$
Cost	$\frac{C_R}{C(Q_R)} = (1-\alpha) \left[\frac{s(1-\varepsilon)\alpha}{r} \frac{\eta}{\eta} \right]^{\alpha} + \alpha \left[\frac{r}{s} \frac{\eta}{(1-\varepsilon)\alpha} \right]^{1-\alpha}$
Net social welfare	$\frac{(W_R - W_M)}{(W_a - W_M)} = \frac{A}{B}$ <p>Where $A \equiv \left[\frac{1}{1-\varepsilon} \left(\frac{Q_R}{Q_M} \right)^{1-\varepsilon} \left(\frac{1-\varepsilon}{\beta} \right)^{\frac{1-\varepsilon}{\gamma}} - \frac{C_R}{C(Q_R)} \left(\frac{Q_R}{Q_M} \right)^{\beta} \left(\frac{1-\varepsilon}{\beta} \right)^{\frac{\beta}{\gamma}} \right]$</p> $\left[- \frac{1}{1-\varepsilon} \left(\frac{1-\varepsilon}{\beta} \right)^{\frac{1-\varepsilon}{\gamma}} - \left(\frac{1-\varepsilon}{\beta} \right)^{\frac{\beta}{\gamma}} \right]$ <p>and $B \equiv \frac{\varepsilon}{1-\varepsilon} \left[\frac{1}{1-\varepsilon} \left(\frac{1-\varepsilon}{\beta} \right)^{\frac{1-\varepsilon}{\gamma}} - \left(\frac{1-\varepsilon}{\beta} \right)^{\frac{\beta}{\gamma}} \right]$</p>

Note: As the derivation of the ratio $(W_R - W_M)/(W_a - W_M)$ is not detailed in Callen et al. (1976), we briefly explain how it can be reconstructed. The net social welfare W_M and W_a are obtained using the formula: $W = [A/(1-\varepsilon)]Q^{1-\varepsilon} - C(Q)$.

Recall that Q_a is the output such that price equals the average cost: $A(Q_a)^{1-\varepsilon} = C(Q_a)$. So, the net social welfare is: $W_a = P(Q_a)Q_a[\varepsilon/(1-\varepsilon)]$.

Remarking that $Q_M = \sqrt[\gamma]{((1-\varepsilon)/\beta)}Q_a$ and using the relation $A(Q_a)^{1-\varepsilon} = C(Q_a)$, the net social welfare obtained in case of a monopoly is:

$$W_M = P(Q_a)Q_a \cdot \left(\frac{1}{1-\varepsilon} \left(\frac{1-\varepsilon}{\beta} \right)^{\frac{1-\varepsilon}{\gamma}} - \left(\frac{1-\varepsilon}{\beta} \right)^{\frac{\beta}{\gamma}} \right), \quad (20)$$

Under rate-of-return regulation, the net social welfare W_R is defined in (16) and can be rearranged as follows:

$$W_R = \frac{A}{1-\varepsilon} \left(\frac{Q_R}{Q_M} \times \frac{Q_M}{Q_a} \times Q_a \right)^{1-\varepsilon} - \frac{C_R}{C(Q_R)} C \left(\frac{Q_R}{Q_M} \times \frac{Q_M}{Q_a} \times Q_a \right). \quad (21)$$

As the output Q_a is such that $A(Q_a)^{1-\varepsilon} = C(Q_a)$, the net social welfare W_R can be rewritten so as to be directly proportional to the total revenue $P(Q_a)Q_a$ obtained by the firm if average cost pricing is implemented:

$$W_R = P(Q_a)Q_a \cdot \left[\frac{1}{1-\varepsilon} \left(\frac{Q_R}{Q_M} \right)^{1-\varepsilon} \left(\frac{1-\varepsilon}{\beta} \right)^{\frac{1-\varepsilon}{\gamma}} - \frac{C_R}{C(Q_R)} \left(\frac{Q_R}{Q_M} \right)^{\beta} \left(\frac{1-\varepsilon}{\beta} \right)^{\frac{\beta}{\gamma}} \right]. \quad (22)$$

References

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