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# ESTIMATION OF LITHOFACIES PROPORTIONS USING WELL AND WELL TEST DATA\*

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ESTIMATION DES PROPORTIONS LITHOLOGIQUES  
À PARTIR DES DONNÉES DE PUITS ET D'ESSAIS  
DE PUITS

La méthode de simulation gaussienne seuillée et la méthode de simulation séquentielle d'indicatrices sont aujourd'hui couramment utilisées pour générer des modèles lithologiques de réservoirs. Ces méthodes consistent à estimer dans un premier temps les proportions (ou probabilités) de faciès et ensuite, à construire des modèles en faciès par troncature d'une fonction aléatoire gaussienne selon les proportions de faciès (ou par tirage au sort selon les probabilités de faciès). La validité d'un modèle ainsi construit dépend fortement de l'exactitude des proportions (ou probabilités) de faciès estimées. En cas de données de puits peu nombreuses, la prise en compte d'autres sources d'informations (connaissance géologique, sismique, essais de puits et historique de production) peut améliorer l'estimation des proportions.

Dans ce travail, on s'intéresse, en particulier, à l'intégration des données de puits et d'essais de puits dans l'estimation des proportions (ou probabilités) de faciès. Une méthode, appelée krigeage itératif sous contraintes d'agrégation (KISCA), est proposée pour estimer les proportions de faciès dans l'aire d'investigation des essais de puits. KISCA consiste à "kriger" conjointement les proportions de tous les faciès dans une aire d'investigation de façon à ce que la perméabilité apparente de l'essai de puits soit respectée via une formule de moyenne en puissance des perméabilités absolues des faciès. En cas d'essais de puits multiples, une procédure itérative est utilisée afin de prendre en compte leur interaction. De simples exemples numériques sont présentés pour illustrer les comportements de la méthode proposée. Pour tester la validité de cette méthode, un réservoir synthétique en faciès est construit et une simulation d'essai de puits est effectuée. La comparaison entre les proportions expérimentales et estimées montre la validité de la méthode proposée. La stabilité numérique et le temps de calcul de cette méthode est comparable au krigeage classique. Les corrélations entre faciès sont partiellement prises en compte par l'introduction des contraintes d'essais de puits et la contrainte de normalité dans le système de krigeage. Les proportions de faciès dans les aires d'investigation des essais de puits peuvent être plus facilement intégrées dans un modèle lithologique construit par simulation gaussienne seuillée ou par simulation séquentielle d'indicatrices.

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ESTIMATION OF LITHOFACIES PROPORTIONS USING  
WELL AND WELL TEST DATA

A crucial step of the two commonly used geostatistical methods for modeling heterogeneous reservoirs: the sequential indicator

simulation and the truncated Gaussian simulation is the estimation of the lithofacies local proportion (or probability density) functions. Well-test derived permeabilities show good correlation with lithofacies proportions around wells. Integrating well and well-test data in estimating lithofacies proportions could permit the building of more realistic models of reservoir heterogeneity. However this integration is difficult because of the different natures and measurement scales of these two types of data.

This paper presents a two step approach to integrating well and well-test data into heterogeneous reservoir modeling. First lithofacies proportions in well-test investigation areas are estimated using a new kriging algorithm called KISCA. KISCA consists in kriging jointly the proportions of all lithofacies in a well-test investigation area so that the corresponding well-test derived permeability is respected through a weighted power averaging of lithofacies permeabilities. For multiple well-tests, an iterative process is used in KISCA to account for their interaction. After this, the estimated proportions are combined with lithofacies indicators at wells for estimating proportion (or probability density) functions over the entire reservoir field using a classical kriging method.

Some numerical examples were considered to test the proposed method for estimating lithofacies proportions. In addition, a synthetic lithofacies reservoir model was generated and a well-test simulation was performed. The comparison between the experimental and estimated proportions in the well-test investigation area demonstrates the validity of the proposed method.

#### EVALUACIÓN DE LAS PROPORCIONES LITOLÓGICAS A PARTIR DE LOS DATOS DE POZOS Y DE PRUEBAS DE POZOS

El método de simulación gaussiana por umbrales y el método de simulación secuencial de indicadores se utiliza corrientemente en la actualidad para generar modelos litológicos de los yacimientos. Estos métodos consisten, en primer lugar, en evaluar las proporciones (o probabilidades) de facies y, a continuación, construir modelos en facies por truncación de una función aleatoria gaussiana acorde a las proporciones de facies (o por sorteo según las probabilidades de facies). La validez de un modelo construido según tal método depende en grado sumo de la exactitud de las proporciones (o probabilidades) de facies evaluados. En caso de datos de pozo poco numerosos, el hecho de tener en cuenta otras fuentes de informaciones (conocimiento geológico, sísmica, pruebas de pozos e historial de producción) puede mejorar la evaluación de las proporciones.

Los autores se han interesado, fundamentalmente, en el presente trabajo, por la integración de los datos de pozo y de pruebas de pozo al proceder a la evaluación de las proporciones (o probabilidades) de facies. Un método, denominado de krigeado iterativo bajo tensiones de agregación (KISCA), se propone para evaluar las proporciones de todos los facies en el área de investigación de las pruebas de pozo. KISCA consiste en krigear conjuntamente las proporciones de todos los facies en un área de investigación, con objeto de que la permeabilidad aparente de la prueba de pozo sea respetada por medio de una fórmula de promedios en potencia de permeabilidad absoluta de facies. En

caso de pruebas de pozos múltiples, se utiliza un procedimiento iterativo con objeto de tener debidamente en cuenta su interacción. Se presentan algunos ejemplos numéricos sencillos para ilustrar el comportamiento del método propuesto. Para someter a prueba la validez de este método, se ha construido un yacimiento sintético en facies y se ha efectuado una simulación de prueba de pozo. La comparación entre las proporciones experimentales y aquellas evaluadas permite demostrar la validez del método propuesto. La estabilidad digital y el tiempo de cálculo de semejante método es comparable con el krigeado convencional. Las correlaciones entre facies son tenidas en cuenta parcialmente por la introducción de tensiones de pozo y la tensión de normalidad en el sistema de krigeado. Las proporciones de facies en las áreas de investigación de pruebas de pozo pueden ser integradas con mayor facilidad en un modelo litológico construido por simulación gaussiana por umbrales o por simulación secuencial de indicadores.

## INTRODUCTION

Recent research on stochastic reservoir modeling constrained by well-test data has been focusing on the approach based on the Bayesian inversion theory and the Markov chain Monte Carlo methods [1, 2, 3]. This approach is highly attractive for it can directly deal with well pressure data rather than well-test derived permeability data and also for it can be extended to history matching. However, it is limited to gross grid reservoir models in the context of continuous Gaussian-related variables (say lognormal permeability field). In the case of a stabilized well-test, it is possible to define an effective permeability in the corresponding investigation area (well-test derived permeability). A method based on simulated annealing has been used for conditioning permeability field to well-test derived permeabilities [4]. Although the annealing process does not call for fluid flow simulations, this method can still be very slow. This paper proposes an alternative approach for incorporating well-test derived permeabilities into lithological reservoir models defined on fine grids.

Consider the two commonly used geostatistical methods: the truncated Gaussian simulation [5] and the sequential indicator simulation [6] for building reservoir lithological models. These methods consist in first estimating the local proportion functions (or probability density functions (pdf)) of lithofacies. Then, the lithofacies model is built by truncating a Gaussian random function with the proportion functions (or by randomly drawing lithofacies from the pdf). A realistic modeling of lithofacies distribution using the above geostatistical methods depends greatly on the accuracy of the estimation of the lithofacies proportions functions (or pdf). In the case of few well data, the incorporation of other sources of information (including geological knowledge, seismic information, well-test data and field production data) would significantly improve the estimation of proportion functions (or pdf).

This paper covers the problem of incorporating well and well-test derived permeability data into the estimation of lithofacies proportion functions (or pdf). A two step approach is used: first the lithofacies proportions in well-test investigation areas are estimated using a new kriging algorithm called KISCA, then these estimates are combined with lithofacies indicators at wells for estimating lithofacies proportion functions (or pdf) over the entire reservoir

field using a classical kriging method. Another method based on the cokriging technique for integrating well and well-test derived permeability data is described in [7]. Also, there are existing methods for incorporating well and seismic data for estimating lithofacies proportion functions [8].

## 1 WELL AND WELL-TEST DATA

The data set is made of a lithofacies description at available wells and a number of well-test derived permeability values. The continuous lithofacies description on the wells is regularly discretized with a fineness defined according to the lithofacies variability along wells. At each point  $x_\alpha$  of the well discretization, an indicator is defined for each lithofacies:

$$I_n(x_\alpha) = \begin{cases} 1 & \text{if } x_\alpha \in \text{lithofacies}_n \\ 0 & \text{otherwise} \end{cases}$$

We consider that the covariance  $C_n(\vec{h})$  of each indicator can be inferred from the well data or other sources of information (e.g. analogous outcrop data).

For each well-test derived permeability  $k_{wt}$ , the investigation area  $V$  is determined and a power averaging formula is adopted to relate the well-test derived permeability to the lithofacies permeabilities.

$$k_{wt}^\omega(V) = \sum_n P_n(V) k_n^\omega \quad (1)$$

where  $k_n$  stands for the permeability of lithofacies  $n$  and  $P_n(V)$  its proportion in  $V$ . The averaging power  $\omega$  is calibrated for each well-test [9] and [10] and  $P_n(V)$  are to be estimated by the method described below.

## 2 ESTIMATION OF LITHOFACIES PROPORTIONS AROUND WELLS

### 2.1 Iterative kriging under aggregation constraints (KISCA)

Let  $N_f$  be the number of lithofacies,  $N_w$  the number of well data and  $N_{wt}$  the number of well-test derived permeability data. For a given investigation area  $V$ ,  $P_n(V)$  ( $n = 1, 2, \dots, N_f$ ) are estimated, at the initial step, by kriging under the normality constraint

$$\sum_n P_n(V) = 1$$

and the well-test constraint (1). The kriging estimator of  $P_n(V)$  is written:

$$P_n^0(V) = \sum_{\alpha} \lambda_n^{\alpha} I_n(x_{\alpha}) \quad \forall n$$

Because the constraints act jointly on all lithofacies,  $P_n(V)$  ( $n = 1, 2, \dots, N_f$ ) cannot be estimated separately. Consequently a "joint" kriging system must be built. This is done by minimizing the sum of the mean square estimation errors of the proportions of all lithofacies (instead of their individual minimization) under the normality and the well-test constraints. The following kriging system is obtained:

$$\left\{ \begin{array}{l} \sum_{\beta} \lambda_n^{\beta} C_n(x_{\alpha}, x_{\beta}) + \mu_n \\ + v_1 I_n(x_{\alpha}) + v_2 I_n(x_{\alpha}) k_n^{\omega} = \overline{C}_n(x_{\alpha}, V) \quad \forall \alpha \quad \forall n \\ \sum_{\alpha} \lambda_n^{\alpha} = 1 \quad \forall n \\ \sum_n \sum_{\alpha} \lambda_n^{\alpha} I_n(x_{\alpha}) = 1 \\ \sum_n \sum_{\alpha} \lambda_n^{\alpha} I_n(x_{\alpha}) k_n^{\omega} = k_{wt}^{\omega}(V) \end{array} \right. \quad (2)$$

which is composed of  $(N_w + 1) * N_f + 2$  linear equations. In terms of computing time, the resolution of this system is nearly equivalent to that of the  $N_f$  individual kriging systems when the proportions of all lithofacies are estimated separately. This is due to the diagonalization of these individual systems in the larger system (2).

Note that at the initial step, only the well-test derived permeability in  $V$  is used for estimating the lithofacies proportions in  $V$ . To account for the interaction between different well-tests, an iterative process is introduced: at step  $l$  ( $> 0$ ), the kriging estimator of the proportion of lithofacies  $n$  in  $V_i$  is written:

$$P_n^l(V_i) = \sum_{\alpha} \lambda_n^{\alpha} I_n(x_{\alpha}) + \sum_{j \neq i} \lambda_n^j P_n^{l-1}(V_j) \quad \forall n$$

Like the initial step, for a fixed  $V_i$ ,  $P_n^l(V_i)$  ( $n = 1, 2, \dots, N_f$ ) are obtained jointly in order to respect the normality

and the well-test constraints. At each step, the kriging weights satisfy the following system:

$$\left\{ \begin{array}{l} \sum_{\beta} \lambda_n^{\beta} C_n(x_{\alpha}, x_{\beta}) + \sum_{k \neq i} \lambda_n^k \overline{C}_n(x_{\alpha}, V_k) + \mu_n \\ + v_1 I_n(x_{\alpha}) + v_2 I_n(x_{\alpha}) k_n^{\omega} = \overline{C}_n(x_{\alpha}, V_i) \quad \forall \alpha \quad \forall n \\ \sum_{\beta} \lambda_n^{\beta} \overline{C}_n(V_j, x_{\beta}) + \sum_{k \neq i} \lambda_n^k \overline{C}_n(V_j, V_k) + \mu_n \\ + v_1 P_n^{l-1}(V_j) + v_2 P_n^{l-1}(V_j) k_n^{\omega} = \overline{C}_n(V_j, V_i) \quad \forall j \neq i \quad \forall n \\ \sum_{\alpha} \lambda_n^{\alpha} + \sum_{j \neq i} \lambda_n^j = 1 \quad \forall n \\ \sum_n \left( \sum_{\alpha} \lambda_n^{\alpha} I_n(x_{\alpha}) + \sum_{j \neq i} \lambda_n^j P_n^{l-1}(V_j) \right) = 1 \\ \sum_n \left( \sum_{\alpha} \lambda_n^{\alpha} I_n(x_{\alpha}) + \sum_{j \neq i} \lambda_n^j P_n^{l-1}(V_j) \right) k_n^{\omega} = k_{wt}^{\omega}(V_i) \end{array} \right.$$

The resolution of this system is quite similar to that of (2) except that the above must be solved at each iteration. However, note that at each iteration, the left hand kriging matrix remains the same only except for the last two lines and columns which are related to the normality and the well-test constraints. This means that only a few more operations are required for the kriging matrix inversion at each new iteration.

## 2.2 Numerical behavior of KISCA

Consider first an example of a single well-test on a 2D reservoir model composed of 3 lithofacies (but KISCA is not limited in 2D). The permeability values of lithofacies 1, 2 and 3 are respectively 1000 md, 100 md and 1 md. The dimensions of the reservoir field are 101 x 101 (arbitrary units). There is well at each of the four corners and one at the center of the field. Two wells hit lithofacies 1, two other wells hit lithofacies 2 and the fifth well hits lithofacies 3 (Fig. 1). A well-test was performed at the central well and a mean permeability value of 200 md in an investigation area of radius 10 was derived from its interpretation.

Figure 2a shows the estimated proportions of the 3 lithofacies as functions of the averaging power  $\omega$ . The variogram ranges of the 3 lithofacies are identical and fixed to 10. Highly different proportions can be observed as the permeability averaging power  $\omega$  varies

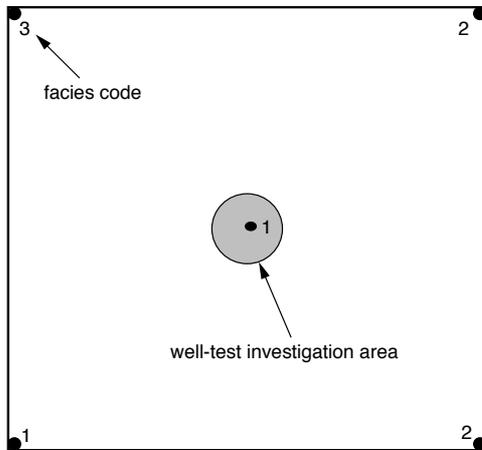


Figure 1

Example of a single well-test on a 2D reservoir model composed of 3 lithofacies: location of the 5 wells and the well-test investigation area.

from  $-1$  (harmonic averaging) to  $1$  (arithmetic averaging), particularly for lithofacies 1 and 3. With a smaller (2) or a larger (50) range, the behavior of the proportion curves remain very similar in shape and with respect to each other (Fig. 2b) and (Fig. 2c).

Figure 3 shows the estimated proportions of the 3 lithofacies as functions of their variogram range with a fixed averaging power  $\omega$ . The proportions of lithofacies 1 and 3 increase as the range varies from 0 to 50 contrary to that of lithofacies 2. Beyond a range of about 50, the proportions of the 3 lithofacies reach their respective sills.

If the variogram ranges of lithofacies 1, 2 and 3 are set respectively to  $50 - a$ ,  $50$  and  $50 + a$ . When parameter  $a$  varies from 0 to 50, the gap between the 3 ranges becomes increasingly important. Figure 4a shows the estimated proportions of the 3 lithofacies as functions of parameter  $a$  with an averaging power  $\omega$  fixed at 0.3. Only slight variation of the 3 proportions can be observed as  $a$  varies from 0 to 50. However, if the well-test constraint is removed, the variation of the proportions increases (Fig. 4b). Furthermore, if both the well-test constraint and the normality constraint are removed, the sum of the 3 estimated proportions becomes more and more different from 1 (Fig. 4c) as  $a$  varies from 0 to 50. In fact, in this last case, we return to classic indicator kriging which respects the normality condition only if the variogram ranges of all lithofacies are identical. Current use of indicator kriging involves a posteriori normalization of the estimated proportions.

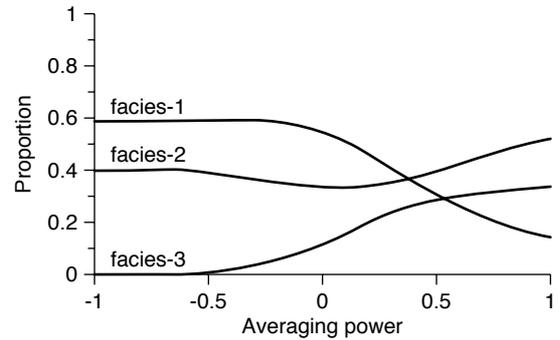


Figure 2a

Estimated proportions of the 3 lithofacies as functions of the averaging power: case with the variogram range = 10.

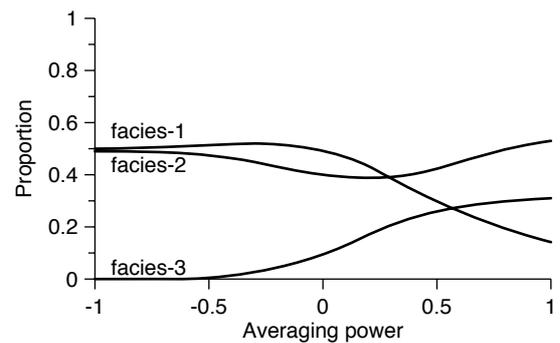


Figure 2b

Estimated proportions of the 3 lithofacies as functions of the averaging power: case with the variogram range = 2.

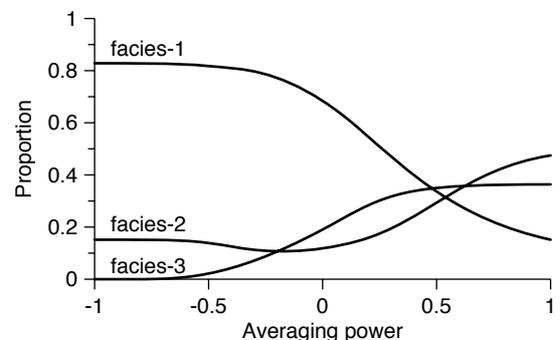


Figure 2c

Estimated proportions of the 3 lithofacies as functions of the averaging power: case with the variogram range = 50.

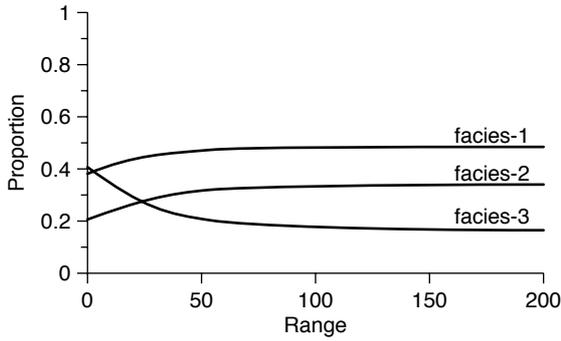


Figure 3  
Estimated proportions of the 3 lithofacies as functions of the variogram range with the averaging power fixed at 0.3.

Figure 4d shows the normalized proportions which are very different from the results obtained by the kriging under the normality constraint (a priori constraint) (Fig. 4b).

The previous example shows the numerical behaviors of the estimated proportions with respect to different parameters using the data from only one well-test. Consider now an example with the interaction of 3 well-tests (Fig. 5). The 3 wells hit respectively lithofacies 1, 2 and 3 which have the same permeabilities as in the previous example. Three permeabilities, 20 md, 200 md and 50 md in respective investigation areas of radius 20, 15 and 10 were derived from the

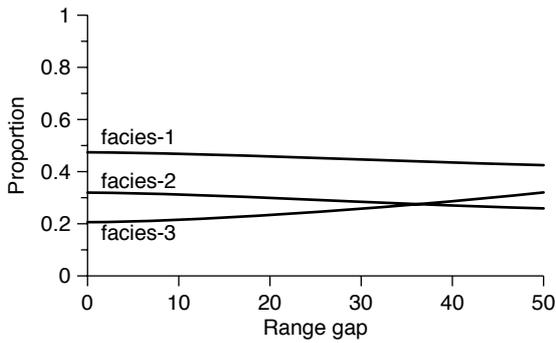


Figure 4a  
Estimated proportions of the 3 lithofacies as functions of the difference between their variogram ranges (the averaging power fixed at 0.3): case with the normality constraint and the well-test constraint.

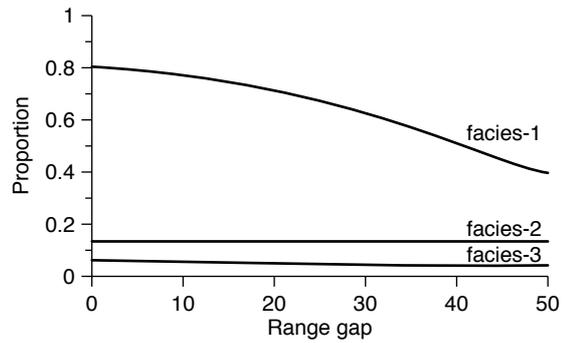


Figure 4c  
Estimated proportions of the 3 lithofacies as functions of the difference between their variogram ranges (the averaging power fixed at 0.3): case without the normality constraint and without the well-test constraint.

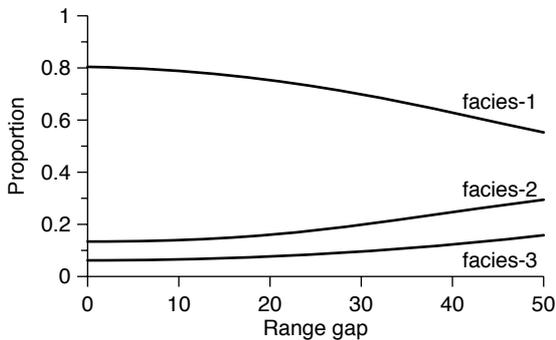


Figure 4b  
Estimated proportions of the 3 lithofacies as functions of the difference between their variogram ranges (the averaging power fixed at 0.3): case with the normality constraint but without the well-test constraint.

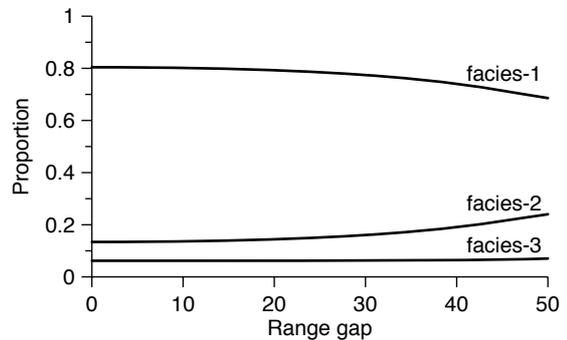


Figure 4d  
Estimated proportions of the 3 lithofacies as functions of the difference between their variogram ranges (the averaging power fixed at 0.3): case with an a posteriori normalization of the estimated proportions but without the well-test constraint.

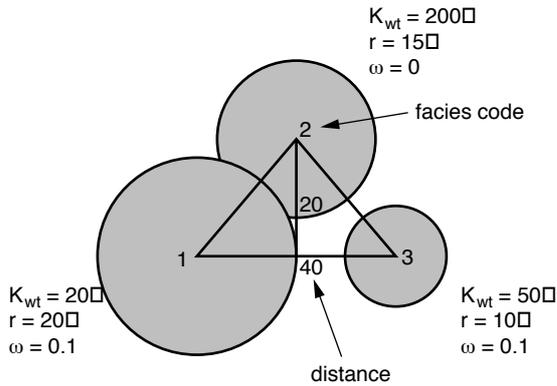


Figure 5

Example of 3 interactive well-tests on a 2D reservoir model composed of 3 lithofacies: location of the 3 wells and the well-test investigation areas.

interpretation of the 3 well-tests. Assume also that different averaging powers are calibrated for the 3 well-tests ( $\omega = -0.1, 0$  and  $0.1$  respectively). The variogram ranges of the 3 lithofacies are identical and equal to 10. Figure 6a shows the estimated proportions in the investigation area of radius 20 as functions of the number of iteration using KISCA. One observes that the estimated proportions reach their respective sills after a few iterations. With a smaller (2) or a larger (50) range, the speed of convergence varies significantly (Fig. 6b and (Fig. 6c).

### 2.3 Validation on a synthetic reservoir

A 2D synthetic reservoir model was built using the truncated Gaussian method (Fig. 7). This model is composed of 3 lithofacies and the field dimensions are  $101 \times 101$  (arbitrary units). The permeability values of lithofacies 1, 2 and 3 are respectively fixed at 50 md, 200 md and 500 md and their variogram ranges are respectively 12, 6 and 15.

A numerical well-test simulation [2] was performed at a well located in the central area of the reservoir field (Fig. 7). The well hits lithofacies 1 which has a much smaller permeability than the two other lithofacies. The interpretation of the well-test simulation gives a permeability value of 156 md in an investigation area of radius 31 [11]. The fact that this well-test derived permeability is much larger than that of lithofacies 1 hit by the well indicates that the presence of the two other

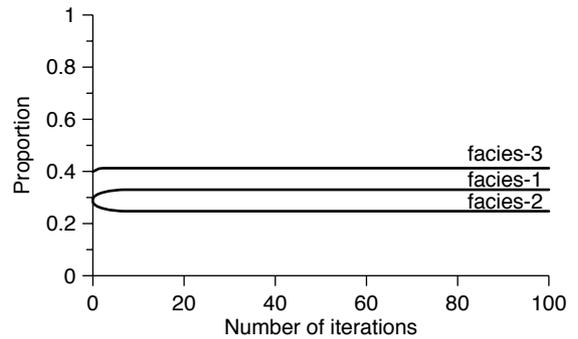


Figure 6a

Estimated proportions of the 3 lithofacies as functions of the number of iterations: case with the variogram range = 10.

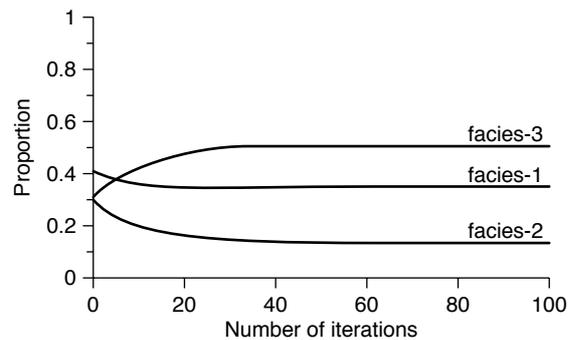


Figure 6b

Estimated proportions of the 3 lithofacies as functions of the number of iterations: case with the variogram range = 2.

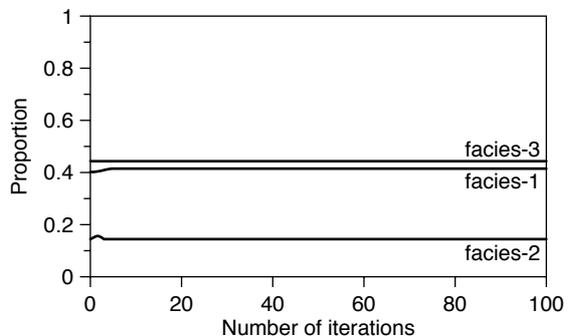


Figure 6c

Estimated proportions of the 3 lithofacies as functions of the number of iterations: case with the variogram range = 50.

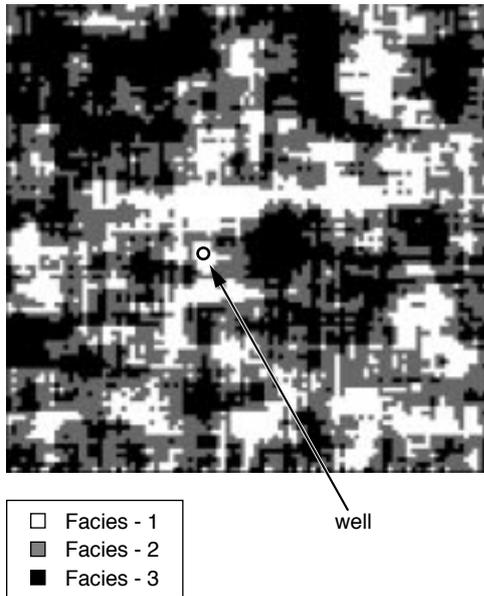


Figure 7

A 2D synthetic reservoir model composed of 3 lithofacies and the location of the tested well.

lithofacies with greater permeabilities is quite high within the investigation area. Fig. 8a, 8b and 8c show the proportions of the 3 lithofacies in the investigation area estimated by KISCA as functions of the averaging power. The experimental proportions in the investigation area of the 3 lithofacies are also indicated in these figures. A good approximation of the experimental proportions is obtained with an averaging power close to 0 (geometric averaging). As a comparison, the kriging with the normality constraint but without the well-test constraint is also used to estimate the lithofacies proportions in the same investigation area. The triangular diagram in Figure 9 shows the relative differences between the experimental proportions, the proportions estimated by KISCA with an averaging power fixed at -0.1 and the proportions estimated by kriging without the well-test constraint. It can be observed that KISCA gives a better estimation of the 3 proportions than the kriging without the well-test constraint. Note also that under the power averaging constraint with  $\omega = -0.1$ , the point which represents the proportions estimated by KISCA is very close to the projection on the constraint line, of the point representing the experimental proportions. This projection point corresponds to the ideal estimation of the proportions under the power averaging constraint.

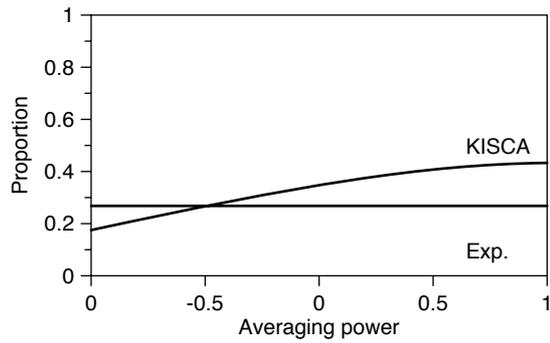


Figure 8a

Estimated proportion of lithofacies 1 as a function of the averaging power: comparison between the experimental proportion and the proportion estimated by KISCA.

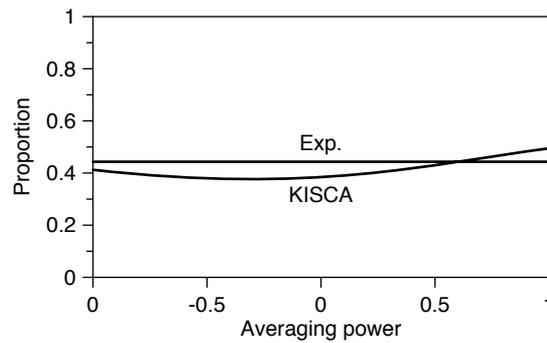


Figure 8b

Estimated proportion of lithofacies 2 as a function of the averaging power: comparison between the experimental proportion and the proportion estimated by KISCA.

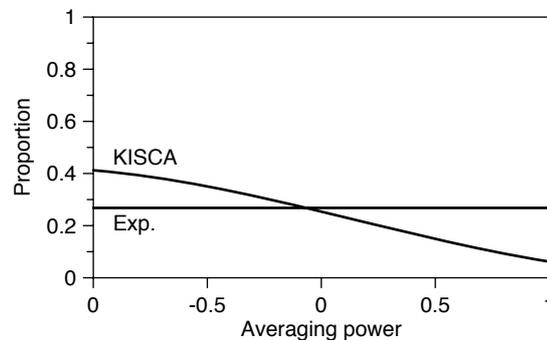


Figure 8c

Estimated proportion of lithofacies 3 as a function of the averaging power: comparison between the experimental proportion and the proportion estimated by KISCA.

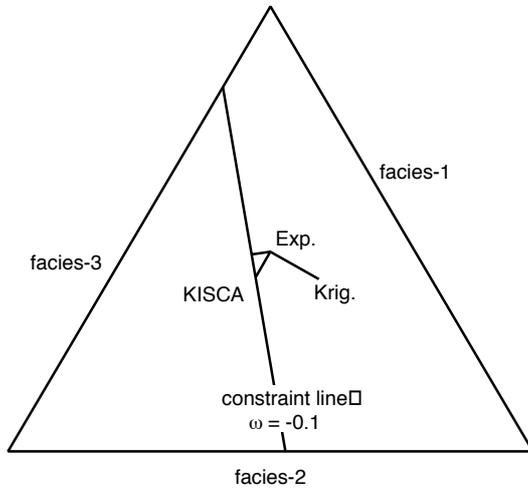


Figure 9

Comparison between the experimental and the estimated proportions via the triangular diagram of proportions: the triangle is equilateral with height equal to 1. The 3 coordinates of each point inside the triangle is defined by the distance of the point from each side. These coordinates represent 3 proportions (the sum of which is equal to 1).

### 2.4 Estimation of 3D proportion functions (or pdf)

Let  $S_x$  be a support located at  $x$  and  $P_n(S_x)$  the proportion of lithofacies  $n$  in  $S_x$ .  $P_n(S_x)$  is a function of which we call proportion function. By using the indicator data of lithofacies  $n$  at wells  $I_n(x_\alpha)$  ( $\alpha = 1, 2, \dots, N_w$ ) and the proportions of lithofacies  $n$  in well-test investigation areas estimated by KISCA  $P_n(V_i)$  ( $i = 1, 2, \dots, N_{wt}$ ), the kriging of  $P_n(S_x)$  is written:

$$P_n^*(S_x) = \sum_{\alpha} \lambda_n^{\alpha} I_n(x_{\alpha}) + \sum_i \lambda_n^i P_n(V_i) \quad \forall n$$

Similar to KISCA,  $P_n(V_i)$  ( $i = 1, 2, \dots, N_{wt}$ ) are jointly estimated in order to respect the normality constraint. Kriging weights  $\lambda_n^{\alpha}$  and  $\lambda_n^i$  satisfy the system:

$$\left\{ \begin{array}{l} \sum_{\beta} \lambda_n^{\beta} C_n(x_{\alpha}, x_{\beta}) + \sum_j \lambda_n^j \overline{C}_n(x_{\alpha}, V_j) + \mu_n \\ + \nu I_n(x_{\alpha}) = \overline{C}_n(x_{\alpha}, S_x) \quad \forall \alpha \quad \forall n \\ \sum_{\beta} \lambda_n^{\beta} \overline{C}_n(V_i, x_{\beta}) + \sum_j \lambda_n^j \overline{C}_n(V_i, V_j) + \mu_n \\ + \nu P_n(V_i) = \overline{C}_n(V_i, S_x) \quad \forall i \quad \forall n \\ \sum_{\alpha} \lambda_n^{\alpha} + \sum_i \lambda_n^i = 1 \quad \forall n \\ \sum_n \left( \sum_{\alpha} \lambda_n^{\alpha} I_n(x_{\alpha}) + \sum_i \lambda_n^i P_n(V_i) \right) = 1 \end{array} \right.$$

Particularly, if  $S_x$  reduces to the point  $x$ , then  $P_n^*(S_x)$  can be interpreted as the probability for lithofacies  $n$  to be present at point  $x$ .  $P_n^*(S_x)$  ( $n = 1, 2, \dots, N_p$ ) constitute a discrete probability system (pdf).

The estimation of lithofacies proportion functions or pdf is an important step in building lithofacies models using the truncated Gaussian method or using the sequential indicator simulation method. The above method permits the incorporation of well and well-test data into estimating lithofacies proportion functions (or pdf). Future work will consist in incorporating all 3 types of information (well data, seismic information and well-test derived proportions) into estimating lithofacies proportion functions (or pdf).

### CONCLUSIONS AND DISCUSSION

In this paper, a new kriging algorithm was presented to estimate lithofacies proportions in well-test investigation areas using well data and well-test derived permeability data. This method consists in kriging jointly the proportions of all lithofacies in a well-test investigation area so that the corresponding well-test derived permeability is respected through a weighted power averaging of lithofacies permeabilities. For multiple well-tests, an iterative process is used to account for their interaction. Some numerical examples were considered to illustrate the behavior of the proposed method. Also, a synthetic lithofacies reservoir model was generated and a well-test simulation was performed. The comparison between the experimental and the estimated proportions in the well-test

investigation area shows the validity of the proposed method. The numerical stability and the computing time of this method are comparable to the classical ordinary kriging method as the correlation between different lithofacies are taken into account through the normality and the well-test constraints rather than through the use of cross-covariance functions.

The lithofacies proportions values derived from well-test data using the above method can be put to use in a number of circumstances. The well-test derived proportions can be used to evaluate the lithological formation around wells. Combined with the lithofacies indicator data at wells, the well-test derived proportions can also be used to estimate the 3D proportion functions required by the truncated Gaussian simulation of lithofacies distribution or to estimate the probability density functions of lithofacies used in the sequential indicator simulation of lithofacies distribution. In addition, the well-test derived proportions can be combined with seismic derived information for estimating 3D lithofacies proportion functions (or pdf).

Note that the above geostatistical methods for building reservoir lithological models reproduce the estimated local proportions only in average. Nevertheless, when the local domain is significantly large with respect to the variogram ranges of lithofacies (this is the case of stabilized well-tests), a good reproduction of local proportions can be obtained for an individual realization. Unlike within a continuous Gaussian-related framework, reproducing local proportions for an individual realization in general cases is a difficult task because of the discontinuity of lithological models. But we think this could be easier than directly reproducing well-test derived permeability data due to the linear relationship between lithofacies indicators and their local proportions. This is a topic of future research.

## NOMENCLATURE

$a$	gap between two variogram ranges of lithofacies
$C_n$	covariance of the indicator function of lithofacies $n$
$C_n$	mean covariance
$\vec{h}$	vector in 3D
$i, j, k$	indices of well-test data
$I_n$	indicator of lithofacies $n$

$k_n$	permeability of lithofacies $n$
$k_{wt}$	well-test derived permeability
$l$	iteration index
$n$	lithofacies index
$N_f$	number of lithofacies
$N_w$	number of well data
$N_{wt}$	number of well-test derived permeability data
$P_n$	proportion of lithofacies $n$
$S$	support in which a lithofacies proportion is defined
$V$	well-test investigation area
$x$	point in 3D
$\alpha, \beta$	indices of well data
$\lambda$	kriging weight
$\mu, \nu$	Lagrangean
$\omega$	permeability averaging power.

## Subscripts

$md$	minidarcy
pdf	probability density function
$wt$	well-test.

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## REFERENCES

- 1 Reynolds A.C., He N., Chu L., and Oliver D.S. (1995) Reparametrization techniques for generating reservoir descriptions conditioned to variograms and well-test pressure data. *Paper SPE 30588* presented at the *SPE Annual Technical Conference and Exhibition*, Dallas.
- 2 Blanc G., Guérillot D., Rahon D. and Roggero F. (1996) Building geostatistical models constrained by dynamic data - a posteriori constraints. *Paper SPE 35478* presented at the *NPF/SPE European 3D Reservoir Modelling Conference*, Stavanger, Norway.
- 3 Oliver D.S., Cunha L.B. and Reynolds A.C. (1997), Markov chain Monte Carlo methods for conditioning a permeability field to pressure data. *Math. Geology*, 29, 1, 61-91.
- 4 Deutsch C.V. (1993) Conditioning reservoir models to well test information, in Soares, A. (ed.), *Geostatistics Troia'92*, 505-518, Kluwer Academic Pub.

- 5 Matheron G., Beucher H., de Fouquet C., Galli A., and Ravenne C. (1987) Conditional simulation of the geometry of fluvio-deltaic reservoirs. *Paper SPE 16753* presented at the *SPE Annual Technical Conference and Exhibition*, Las Vegas.
- 6 Journé A. Nonparametric estimation of spatial distribution, *Journal of Mathematical Geology*, 15, 3, 445-468.
- 7 Haas A. and Noetinger B. (1996) Stochastic reservoir modelling constrained by well test permeabilities. *Paper* presented at the *5th International Geostatistics Congress*, Wollongong, Australia.
- 8 Moulière D., Beucher H., Hu L.Y., Fournier F., Terdich P., Melchiori F. and Griffi G. (1996) Integration of seismic derived information for reservoir stochastic modeling using truncated Gaussian approach. *Paper* presented at the *5th International Geostatistics Congress*, Wollongong, Australia.
- 9 Alabert F. (1989) Constraining description of randomly heterogeneous reservoirs to pressure test data: a Monte Carlo study. *Paper SPE 19600* presented at the *SPE Annual Technical Conference and Exhibition*, San Antonio.
- 10 Noetinger B. (1993) A pressure moment approach for helping pressure transient analysis in complex heterogeneous reservoir. *Paper SPE 26466* presented at the *SPE Annual Technical Conference and Exhibition*, Dallas.

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