Accelerating and Stabilizing the Vapor-Liquid Equilibrium (VLE) Calculation in Compositional Simulation of Unconventional Reservoirs Using Deep Learning Based Flash Calculation

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1 **Accelerating and Stabilizing the Vapor-Liquid Equilibrium (VLE) Calculation in Compositional**

2 **Simulation of Unconventional Reservoirs Using Deep Learning Based Flash Calculation**

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11 **Abstract**

12   The flash calculation with large capillary pressure often turns out to be time-consuming and

13 unstable. Consequently, the compositional simulation of unconventional oil/gas reservoirs, where large

14 capillary pressure exists on the vapor-liquid phase interface due to the narrow pore channel, becomes

15 a challenge to traditional reservoir simulation techniques. In this work, we try to resolve this issue by

16 combining deep learning technology with reservoir simulation. We have developed a compositional

17 simulator that is accelerated and stabilized by stochastically-trained proxy flash calculation.

18   We first randomly generated 300,000 data samples from a standalone physical flash calculator.

19 We have constructed a two-step neural network, in which the first step is the classify the phase

20 condition of the system and the second step is to predict the concentration distribution among the

21 determined phases. Each network consists of four hidden layers in between the input layer and the

22 output layer. The network is trained by Stochastic Gradient Descent (SGD) method with 100 epochs.

23   With given temperature, pressure, feed concentration pore radius, the trained network predicts

24 the phases and concentration distribution in the system with very low computational cost. Our results

25 show that the accuracy of the network is above 97% in the metric of mean absolute percentage error.

26 The predicted result is used as the initial guess of the flash calculation module in the reservoir

27 simulator. With the implementation of the deep learning based flash calculation module, the speed of

28 the simulator has been effectively increased and the stability (in the manner of the ratio of

29 convergence) has been improved as well.

30 **Keywords**
31 Flash calculation; unconventional reservoirs; deep learning; proxy calculation; reservoir simulation

32 **1. Introduction**

33   The oil and gas stored in unconventional reservoirs has some unique transport mechanism and

34 phase behavior, including the pore confine effect [1,2], the large capillary pressure effect [3,4],

35 multiscale pore structures [5] and gas slippage effect [6–9]. Moreover, the prediction of the complex

36 phase behaviors in unconventional reservoirs requires compositional modeling, of which the flash

37 calculation is an essential yet time-consuming portion. To improve the speed as well as the stability of

38 the flash calculation, several techniques have been adopted, including reduction method [10], phase

39 stability test [11] and so on. Recently, the fast-arising artificial intelligence (AI) techniques have drawn

40 the attention of researchers. Particularly, the fast development of the optimization algorithm, as well as

41 hardware infrastructure, have greatly promoted the advance of stochastic learning techniques. The

42 rapid development of GPU enables the training of deep learning (DL) networks (also known as Artificial

43 Neural Network (ANN)) [12]. There are several trials of combining AI with flash calculation. Gaganis et

44 al. [13,14] are among the first to propose the application of neural network approach in developing

45 proxy flash calculation. In their work, support vector classifier (SVC) is used to conduct phase stability

46 test and a single layer ANN network is used to replace the physical flash calculation for liquid-vapor

47 phase equilibrium calculation. Kashinath et al. [15] further improved Gaganis et al.'s model by bringing

48 out a novel framework to conduct an isothermal flash calculation. In their work, the relevance vector

49 machine [16] is combined with an single-layer artificial neural network. The former technique is used to

50 classify phase condition, while the latter is used to determine the concentration distribution. All these

51 proxy models have shown sound accuracy and have been successfully implemented into reservoir

52 simulators, improving the speed of compositional simulation. Moreover, El-Sebakhy [17] and Rafiee-

53 Taghanaki et al. [18] used the support vector machine technique [19–22] to predict the PVT properties

54 of crude oil, including gas oil ratio, oil volume factor, density and so on. Artificial neural networks have

55 also been used to predict the PVT behavior of crude oil [23] and gas mixtures [24]. Nikravesh et al. [25]

56 reviewed the applications of artificial intelligence techniques in the exploration and development of

57 petroleum reservoirs.

58   In this work, we further extend the work listed above to the compositional simulation of

59 unconventional reservoirs with large capillary pressure effect. We have developed a deep-learning

60 based flash calculation module (proxy flash calculation) for the prediction of phase behaviors of oil and

61 gas in unconventional reservoirs. This proxy flash calculation adopts multi-layer fully connected layers

62 to regress the training data. The input parameters of our model include pressure, temperature, feed

63 concentration and pore radius. The accuracy of the network is above 97% in the metric of mean

64 absolute percentage error. The proxy flash calculation module is used as a preconditioner of the

65 physical flash calculation and has been implemented in a reservoir simulator. We have also compared

66 the performance of the network with different number of hidden layers. The novelty of this work lies in

67 the implementation of the deep learning based flash calculation module as a preconditioner for both

68 phase condition detection and concentration determination, which improves the speed as well as the

69 stability of compositional simulation of unconventional reservoirs while maintaining the same results as

70 physical flash calculation.

71   This paper is organized as follows. In Chapter 2, we present the physical flash calculation used
72 to train the network. In Chapter 3, we describe the structure, training as well as results of our deep-
73 learning based flash calculation module. In Chapter 4, we describe the implementation of the
74 developed module into an in-house reservoir simulator. In Chapter 5, we present the results of field
75 scale reservoir simulation. In Chapter 6, we summarize and conclude this work.
76

77 **2. Forward modeling**

78   In this section, we briefly introduce the governing equations and flash calculation module used
79 for the forward modeling.

80   **2.1. Flow governing equations**

81   The reservoir simulator used in this work is named as MSFLOW_CO2 [3,26,27].
82 MSFLOW_CO2 is a general three-dimensional reservoir simulator for the simulation of complex
83 multiphase flow in porous media. Based on the law of mass conservation, the flow governing equations
84 of MSFLOW_CO2 describe the transport of hydrocarbon components in a petroleum reservoir. For a
85 vapor-liquid compositional system with N_C components, the mass conservation equation for component
86 *k* is as below

87 $$\frac{\partial}{\partial t}\left(\phi\sum_{\beta}S_{\beta}\rho_{\beta}x_{\beta}^{k}\right)=\nabla\cdot\left[-K_a\frac{K_{r\beta}\rho_{\beta}}{\mu_{\beta}}\left(\nabla P_{\beta}-\rho_{\beta}\vec{g}\right)x_{\beta}^{k}\right]+q^k,\quad \beta=L,G,\quad k=1,..,N_C \qquad 1$$

88 where ϕ is the rock porosity. β is the phase index referring to the liquid (L) or vapor/gas (G) phase. S_{β}
89 and ρ_{β} are the saturation and density of phase β, respectively. $K_{r\beta}$, μ_{β} and P_{β} are the relative
90 permeability, viscosity and pressure of phase β, respectively. K_a is the apparent permeability. While
91 for the liquid phase, K_a is the same as the rock absolute permeability K_{∞}, for the vapor phase, $K_a=$
92 $K_{\infty}(1+b/p)$, in which *b* is the Klinkenberg parameter. x_{β}^{k} is the mole concentration of component *k* in
93 phase β. $\vec{g}$ is the gravity term and *q* is the sink/source term. In this work the pore compressibility is
94 temporally ignored.

95   In MSFLOW_CO2, Equation 1 is discretized and solved by the Integrated Finite Difference (IFD)
96 method, the details of which can be found in Ref. [28]. The flow between the rock matrix and the
97 fracture system is described by the dual-porosity model [29]. The nonlinear system resulted from the
98 discritization of IFD is solved by Newton-Raphson's approach. Within each nonlinear iteration, the
99 resulted linear system is solved by a multiscale linear solver [30].

100   **2.2. Flash calculation with capillary pressure**

101   In a compositional reservoir simulator, given the pressure (*P*), temperature (*T*) and mole

102   concentration of each component (z_i), the flash calculation module predicts the phase condition as well

103   as the concentration of each component in each phase.

104   In this work, we consider a two-phase system with vapor (*V*) and liquid (*L*) phase. The mole

105   concentration of a component in the vapor (gas) phase and in the oil (liquid) phase is denoted

106   respectively as y_i and x_i. Meanwhile, the total mole concentration of the vapor phase and the oil phase

107   is denoted as n_V and n_L respectively. We then have the following relationship.

108    $z_i = x_i n_L + y_i n_V$ 2

109   The phase behavior of fluids stored in unconventional reservoirs is unlike that in conventional

110   reservoirs. In the narrow pores of unconventional reservoirs, the capillary pressure P_c between phases

111   can be no longer ignored [31,32]. For simplicity, in this work, the capillary pressure, which is the

112   difference between the vapor pressure p^V and the oil phase pressure p^L, is calculated as below,

113   assuming the oil phase is the wetting phase.

114    $P_c = p^V - p^L = \dfrac{2\sigma_{VL}\cos\theta}{r}$ 3

115   In the above equation, σ_{VL} is the interfacial tension between the vapor phase and the liquid phase. θ is

116   the contact angle and *r* is the pore radius.

117   σ_{VL} is calculated using the model from the work of Macleod [33] and Sugden [34], as follows

118    $\sigma_{VL} = \left(P_a^L \rho^L - P_a^V \rho^V \right)^{\nu} = \left(\rho^L \sum_{i=1}^{N_C} x_i P_{a,i} - \rho^V \sum_{i=1}^{N_C} y_i P_{a,i} \right)^{\nu}$ 4

119   where ρ^L and ρ^V is the molar density of the liquid phase and the vapor phase respectively. P_a^L and P_a^V

120   is the parachor for the liquid phase and the vapor phase respectively. $P_{a,i}$ is the parachor of component

121   *i* , the value of which is listed in Table A.2. ν is a parameter that is by default set to be 3.6 [35].

122   At the equilibrium condition, the fugacity of component *i* in the vapor phase f_i^{v} and in the liquid

123   phase f_i^{l} should be equal, as

124    $f_i^{v} = f_i^{l}$ 5

125   We introduce the fugacity coefficient of component *i* in the vapor phase and the liquid phase, as

126 $\quad \Phi_i^V = \dfrac{f_i^V}{y_i p^V}$ 6

127 $\quad \Phi_i^L = \dfrac{f_i^L}{x_i p^L}$ 7

128 Moreover, the equilibrium ratio is defined as

129 $\quad K_i = \dfrac{y_i}{x_i} = \dfrac{f_i^V/\left(\Phi_i^V p^V\right)}{f_i^L/\left(\Phi_i^L p^L\right)} = \dfrac{\Phi_i^L p^L}{\Phi_i^V\left(p^L + P_c\right)}$ 8

130 By combining Equation 5 and Equation 8 and considering the constraints that $\displaystyle\sum_{i=1}^{N_C} z_i = \sum_{i=1}^{N_C} y_i = \sum_{i=1}^{N_C} x_i = 1$,

131 we can derive the Rachford-Rice equation as

132 $\quad \displaystyle\sum_{i=1}^{N_C} \dfrac{z_i\left(K_i - 1\right)}{n_L + K_i\left(1 - n_L\right)} = 0$ 9

133 In this work, Peng-Robin equation of state (PR-EOS) [36] without volume factor correction is used for

134 the calculation of the PVT properties of oil and gas. PR-EOS is a widely adopted cubic equation of

135 state. For phase β (vapor or liquid), the compressibility can be formulated as

136 $\quad \left(Z^\beta\right)^3 + \left(B^\beta - 1\right)\left(Z^\beta\right)^2 + \left(A^\beta - 3\left(B^\beta\right)^2 - 2B^\beta\right)Z^\beta - \left(A^\beta B^\beta - \left(B^\beta\right)^2 - \left(B^\beta\right)^3\right) = 0 \ldots..\beta = L,V$ 10

137 where the term A^β and B^β are defined as follows

138 $\quad A^\beta = \dfrac{a_m p^\beta}{R^2 T^2}$ 11

139 $\quad B^\beta = \dfrac{b_m p^\beta}{RT}$

140 12

141 In the above equations, Z^β is the compressibility of phase β. the terms a_m and b_m are defined as

142 follows as

143 $\quad a_m = \displaystyle\sum_{i=1}^{N_C}\sum_{j=1}^{N_C} z_i z_j a_{ij}$ 13

144
145 $\quad a_{ij} = \left(1 - k_{ij}\right)\sqrt{\alpha_i a_i \alpha_j a_j}$ 14

146
147 $\quad a_i = 0.45724\dfrac{R^2 T_{C_i}^2}{P_{C_i}}$ 15

148

149 $$\alpha_i = \left[1 + S_i^* \left(1 - \sqrt{T_{C_i}/T} \right) \right]^2$$ 16

150 $$S_i^* = 0.37464 + 1.54226\omega_i - 0.26992\omega_i^2$$ 17

151 $$b_m = \sum_{i=1}^{N_C} b_i$$ 18

152 $$b_i = 0.0778 \frac{RT_{C_i}}{P_{C_i}}$$ 19

153 where R is the gas constant. ω is the acentric factor. k_{ij} is the binary interaction coefficient between

154 component i and the component j. T_C and P_C is the critical temperature and critical pressure

155 respectively. The critical properties for the hydrocarbon used in this work are from NIST data [37]. The

156 lartest root of the cubic equation is assigned to the compressibility of the vapor phase, while the

157 smallest root is assigned to the compressibility of the liquid phase. Based on the assumption of

158 isothermal flash calculation, in PR-EOS, the fugacity coefficients are calculated as

159 $$\ln\left(\Phi_i^\beta\right) = \frac{b_i\left(Z^\beta - 1\right)}{b_m} - \ln\left(Z^\beta - B^\beta\right) - \frac{A^\beta}{2\sqrt{2}B^\beta}\left(\frac{2\Psi_i}{a_m} - \frac{b_i}{b_m}\right)\ln\left(\frac{Z^\beta + \left(1+\sqrt{2}\right)B^\beta}{Z^\beta - \left(1-\sqrt{2}\right)B^\beta}\right)$$ 20

160 In the above equation, the term Ψ_i is

161 $$\Psi_i = \sum_j x_j \sqrt{a_i a_j \alpha_i \alpha_j}\left(1 - k_{ij}\right)$$ 21

162 The initial guess of the equilibrium ratio K_i^0 is calculated by Wilson's equation

163 $$K_i^0 = \frac{p_{ci}}{p}\exp\left(5.37\left(1+\omega_i\right)\left(1-\frac{T_{ci}}{T}\right)\right)$$ 22

164 Initially, the capillary pressure is set to be 0.

165   Prior to the flash calculation, a Gibbs energy based phase stability test is performed to

166 preliminarily determine the single phase region. The approach used here follows the work of Sherafati

167 and Jessen [35]. The tangent plane distance (TPD) based on Michelsen's formulation [38] is as follows

168 $$TPD\left(\mathbf{z}_T\right) = \sum_{i=1}^{N_C} z_{Ti}\left[\mu_i^T\left(\mathbf{z}_T, P^T\right) - \mu_i^z\left(\mathbf{z}, P^z\right)\right]$$ 23

169 where $\mathbf{z}$ and $\mathbf{z}_T$ refers to the mole concentration of the feed and a trial phase, respectively. μ_i^T and μ_i^z is

170 the fugacity of component i in the trial phase and the feed respectively. P^T and P^z is the trial phase

171 pressure and the reference pressure respectively. By switching the variable $\ln Z_i^T = \ln z_i^T - k$ where k is

172 the reduced value of the tangent plane distance at the stational points of Equation 23, the above

173 equation can be expressed as

174 $$TPD\left(\mathbf{Z}_T\right)=1+\sum_{i=1}^{N_C}Z_{Ti}\left[\ln Z_{Ti}+\ln\varphi_i^T+\ln P^T-\left(\ln z_i+\ln\varphi_i+\ln P^z\right)-1\right]$$ 24

175 where φ_i and φ_i^T denotes the fugacity coefficient of component *i* in the feed and the trial phase,

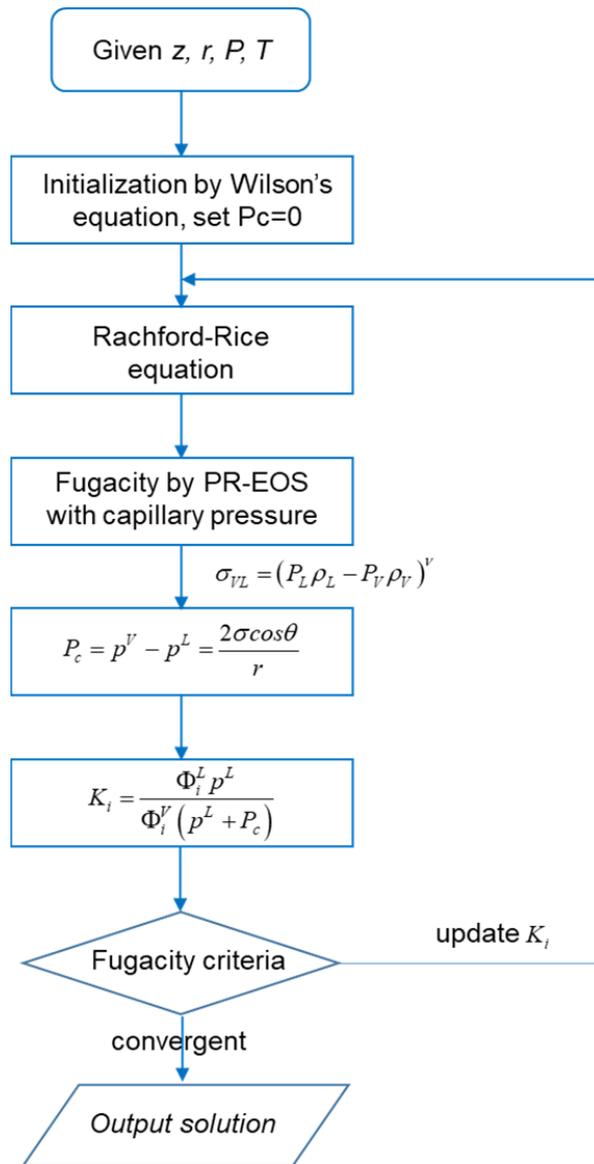176 respectively. Finding the stationary points of TPD is equivalent to solving the below equation

177 $$\ln Z_{Ti}+\ln\varphi_i^T+\ln P^T-\left(\ln z_i+\ln\varphi_i+\ln P^z\right)=0$$ 25

178 The above equation is iteratively solved by an accelerated direct substitution approach based on

179 dominant eigenvalue method, as described by Orbach and Crowe [39]. It should be noticed that, unlike

180 the phase stability test without capillary pressure, the solution of Equation 25 should take the pressure

181 difference between the trial phase and the feed into consideration, as $P^T-P^Z=\pm P_C$. The sign before

182 the capillary pressure is positive if the trial phase is the non-wetting (vapor) phase and the feed is the

183 wetting (liquid) phase. The sign is negative if the trial phase and the feed is wetting phase and non-

184 wetting phase respectively.

185 Based on the solution of the Rachford-Rich equation and the capillary pressure equation, the

186 fugacities, as well as densities of the fluids, are obtained. The algorithm iteratively tunes the solution to

187 minimize the residual of the equilibrium ratio as well as the capillary pressure until certain criteria are

188 satisfied. In this work, the criterion of the convergence of the equilibrium ratio is set as

189 $$\sum_i\left|\frac{K_i}{K_i^*}-1\right|\leq 1.0e^{-4}$$ 26

190 where K_i^* is the equilibrium ratio solved at the previous iteration step. A flowchart of the K-value based

191 flash calculation with capillary pressure effect is shown in Figure 1. The parameters, including binary

192 interaction factor and parachor values, for the physical flash calculation are listed in Appendix A. As an

193 example, the phase envelops of n-Decane-CO_2 binary mixture with and without the capillary pressure

194 are shown in Figure 2. Our results match well with experimental results [40].

195

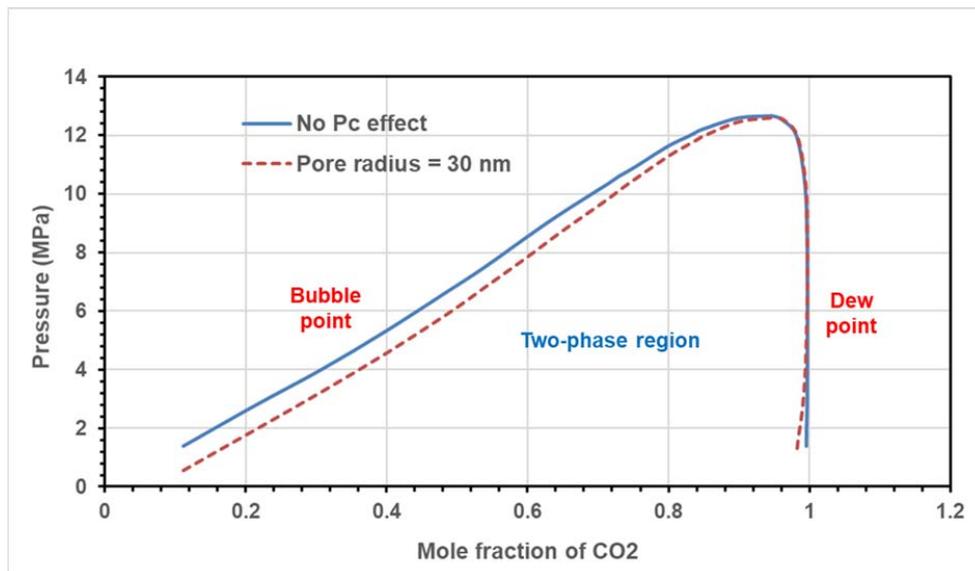**Figure 1 Flowchart of the K-value based flash calculation with capillary pressure effect.**

196

197

198

200 **Figure 2 Phase envelop of n-Decane-CO_2 binary mixture at 71 °C.**
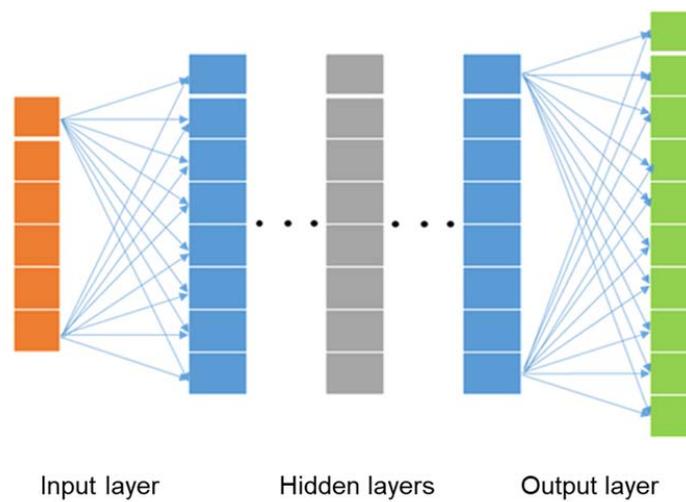
201
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203 **3. Proxy flash calculator**

204 **3.1. General information**

205   In this work, we have developed a data-driven flash calculation module based on deep learning

206 techniques to improve the speed and convergence performance of the flash calculation in the

207 unconventional reservoirs. In the new framework, the initial guess of the flash calculation is obtained

208 from a stochastically trained neural network instead of Wilson's equation. We use deep-learning based

209 stochastic training technique to develop the proxy simulator. We have trained a neural network, which

210 has an input layer, an output layer, and four hidden layers. Within each layer, there are several neurons

211 (elements). All neurons belonged to two neighboring layers are fully connected, as shown in Figure 3.

212   The neural network adopts fully connected (dense) layers and is trained as a standalone

213 module. Once trained, the neural network predicts the phase condition, capillary pressure as well as

214 concentration distribution by simple interpolations. As will be shown in the later chapters, the accuracy

215 of the developed network is above 97%. Therefore, a much more accurate initial guess for the flash

216 calculation can be obtained. The fully connected layers are an imitation of human's neural system.

217 Each element within the network has one weight value and one bias value, indicating the 'contribution'

218 of the element. In the fully connected layers, each element is connected to all elements belonged to its

219 neighboring layers. Activation functions are used between layers. For a given set of input parameters,

220 the network predicts the output results by interpolating from the weight values and the bias values.

221 During the training process, the weight and the bias of the elements are optimized by certain

222 optimization algorithms to achieve the best prediction. Then in the prediction step as shown in Figure 4,
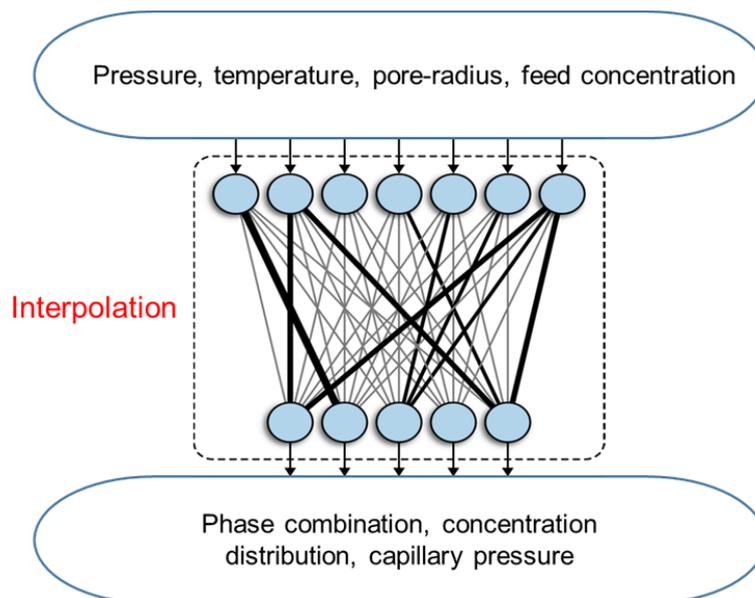
223 the simulator only needs to load the trained weight and bias values into the memory and conduct
224 simple interpolation, which is very cheap in terms of computational time.

225   In this work, the training process consists of two steps. The first step, which is named the phase
226 classification step, determines the phase condition of the system under the given condition. In the
227 second step, which is named the concentration determination step, the phase ratio, component
228 concentration, and capillary pressure are determined. The input parameters for both steps include
229 pressure, temperature, feed concentration and pore radius. The input parameters are all normalized to
230 [0,1] scale before being substituted into the network.

Input layer   Hidden layers   Output layer

231
232   **Figure 3 Conceptual model of the fully connected neural network.**

233
234   **Figure 4 Conceptual framework of the prediction step.**

235

236 **3.2. Phase classification**

237 The network for the phase classification step consists of six layers, including the input layer

238 (layer 1), the output layer (layer 6) and four hidden layers (layer 2 to 5). The number of input

239 parameters is $N_c + 3$. The input parameters x include the feed concentration, pressure, temperature,

240 and pore radius. All input parameters are normalized to [0,1] scale. The dimension of layer 1 to layer 4

241 is 64, and the dimension of the output layer is 3. Therefore, the network classifies the phase condition

242 into three types, namely pure vapor phase (V), pure oil phase (L), and double phases (V+L).

243 $$X = \left[P, T, r, z_1, ..., z_{N_c} \right]^T$$ 27

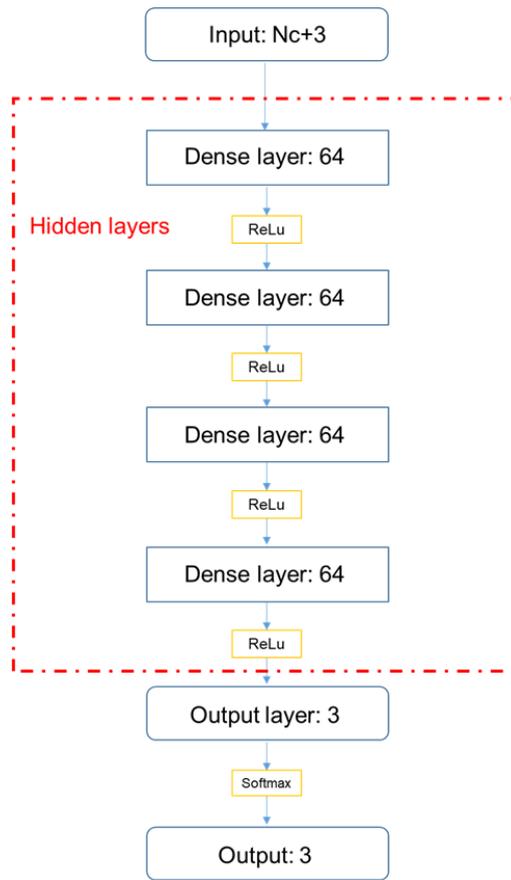244 The activation function for layer 1 to 4 is ReLU function, as follows

245 $$\mathrm{ReLU}(x) = \begin{cases} 0 & \text{for} \quad x < 0 \\ x & \text{for} \quad x \geq 0 \end{cases}$$ 28

246 The activation function for the output layer is Softmax function, as follows

247 $$\mathrm{softmax}(x)_j = \frac{e^{x_j}}{\sum_{k=1}^{K} e^{x_k}}$$ 29

248 where K is the total number of parameters and e^x is the exponential function. A detailed structure of the

249 network is as shown in Figure 5.

250

251

Figure 5 Structure of the fully connected neural network for the phase classification step. The numbers refer to the dimension of the layers.

254

3.3. Concentration determination

The network for the concentration determination step consists of six layers, including the input layer (layer 1), the output layer (layer 6) and four hidden layers (layer 2 to 5). The dimension, as well as physical meanings of the input parameters of this step, are the same as those of the phase classification step. The dimension of layer 1 to layer 4 is 64, and the dimension of the output layer is $2N_c + 3$, including the capillary pressure P_C, the vapor phase ratio n_V, the oil phase ratio n_L, and the component concentration in the vapor phase $y_i, i = 1, ..., N_c$ and oil phase $x_i, i = 1, ..., N_c$. The activation function for layer 1 to 4 is ReLU function, as shown in Equation 28. The activation function for the output layer is Sigmoid equation, as shown in Equation 30.
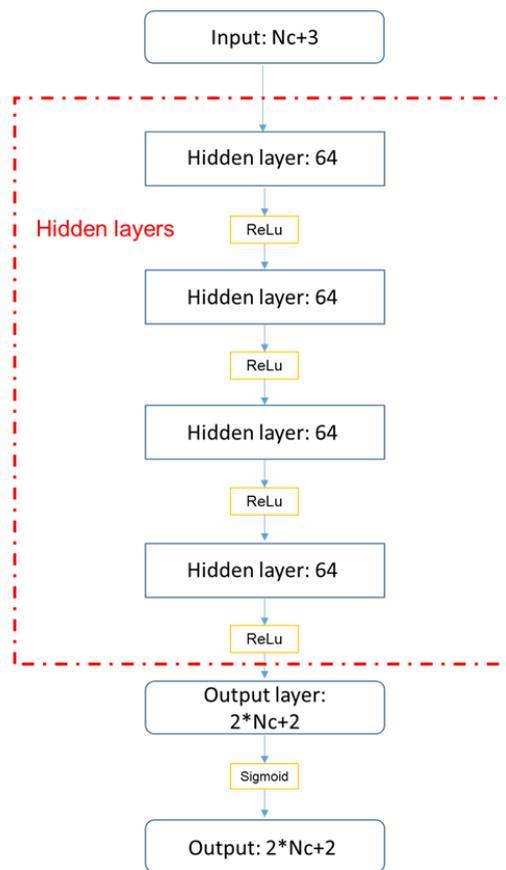
$$\text{sigmoid}(x) = \frac{1}{1 + e^{-x}} \qquad\qquad 30$$

A detailed structure of the network is as shown in Figure 6.

266

Figure 6 Structure of the neural network for the concentration determination step. The numbers refer to the dimension of the layers.

3.4. Training

We have investigated five cases with different combinations of hydrocarbon components. For each case, we use 300,000 training samples generated from the standalone flash calculation module described in Chapter 2. The samples are generated randomly using Latin Hypercube Sampling [41,42] technique. The range of the input parameters is listed in Table 1.

Table 1 Range of the parameters of the training samples.

	Unit	Minimum	Maximum
Pressure	MPa	1	80
Temperature	°C	40	100
Pore radius	nm	30	100
Feed concentration	dimensionless	0	1

280 We use stochastic gradient descent (SGD) algorithm [43] to train the network on Keras [44] with a GTX
281 1080Ti GPU of 11 GB in memory. The algorithm of SGD can be briefly described as follows (Bottou
282 2012). For an object function Q with the primary variable $\mathbf{w}$ of n dimensions,

283 $$\Theta(\mathbf{w}) = \frac{1}{n}\sum_{i=1}^{n}\Theta_i(w_i)$$ 31

284 Instead of optimizing all n dimensions at the same time, SGD randomly optimizes a randomly chosen
285 group (batch) of the variables using gradient descent optimization, as below

286 $$w_{t+1} = w_t - \eta\nabla_j\Theta(w_t)$$ 32

287 where t is the number of iteration steps. j is the index of a batch. η is the learning rate. After the
288 completion of one epoch, all the training samples are shuffled. Hence, the optimization of one high-
289 dimensional problem is effectively converted to the optimization of numerous low-dimensional
290 problems. In this work, we have compared the choice of the learning rate. The optimal value is found to
291 be 0.001. The neural network for the phase classification step and the concentration determination step
292 is trained with 100 epochs, respectively. We use the mean absolute percentage error (MAPE) as a
293 metric (loss function) during the training, which is defined as follows.

294 $$\text{MAPE} = \frac{100\%}{N}\sum_{i=1}^{N}\left|\frac{A_i - F_i}{A_i}\right|$$ 33

295 In the above equation, N refers to the number of testing samples. A and F refer to actual value and
296 predicted (forecast) value, respectively. The accuracy of the training sample is cross validated by K-
297 folds approach. The training samples are randomly divided into ten groups. For every ten epochs of the
298 training, one group is chosen for testing and the rest of the nine groups are used for training. The
299 variation of the loss function during the training process for Case 5 is shown in Figure 7. The error
300 metric for each of the five cases is listed in Table 2. According to the results, the accuracy of our neural
301 network is above 97% for the cases we run. We have compared the accuracy of the network with
302 different number of hidden layers and found out network with four hidden layers achieves optimal
303 performance. Generally speaking, more hidden layers (with more degree of freedom) result in deeper
304 network and better accuracy. However, beyond a certain level the addition of more layers cannot
305 contribute to accuracy and may even cause over-fitting issue [12]. We have also compared the
306 performance of different activation functions for the hidden layers, including sigmoid function (as
307 defined in Equation 30), tanh function [12] and ReLU function (as defined in Equation 28) for Case 4.
308 The results are listed in Figure 9, which shows that ReLU function achieves the highest accuracy.
309 However till today the choice of activation function as well as the number of layers is still like an art.
310 Moreover, as expected when the number of hydrocarbon components increases, the accuracy
311 decreases accordingly, due to the increase of the dimensions in the parameter space. In general, the
312 proxy flash calculator based on deep learning techniques is much more accurate than any other

313 existing techniques.

314

315

316 **Table 2 Summary of errors of the fully connected network.**

Case index	Feed component	Phase Classification	Concentration Determination	Overall Accuracy
		Mean Absolute Percentage Error	Mean Absolute Percentage Error	Mean Absolute Percentage Accuracy
1	C1+C2+C3	0.01%	0.46%	99.53%
2	C1+C2+C3+C6	0.02%	0.73%	99.25%
3	C1+C2+C3+C4+C5	0.02%	1.02%	98.96%
4	C1+C2+C3+C5+C7+C9	0.04%	1.86%	98.10%
5	C1+C2+C3+C4+C5 C8+C9+C10	0.06%	2.24%	97.83%

317

318

319

320 **Figure 7 Variation of the loss function of the phase classification step during the training**
321 **process of Case 4.**

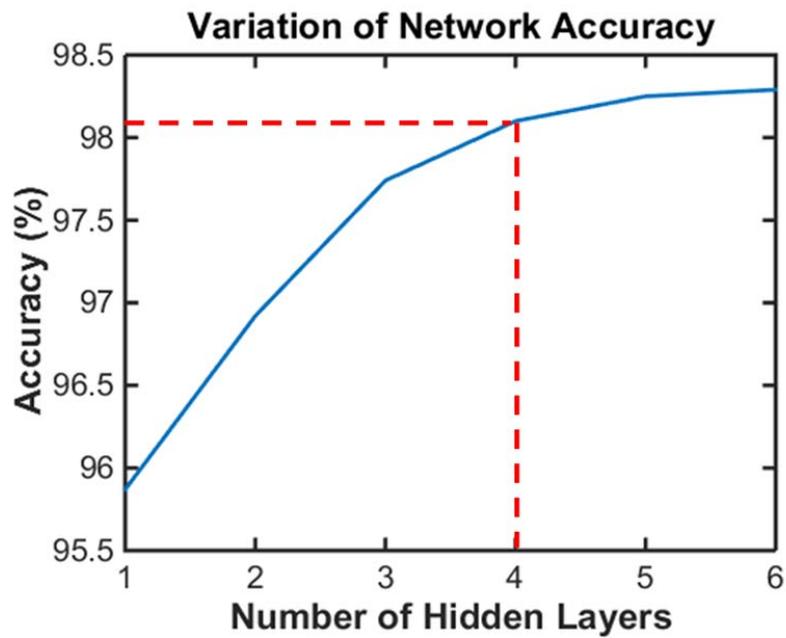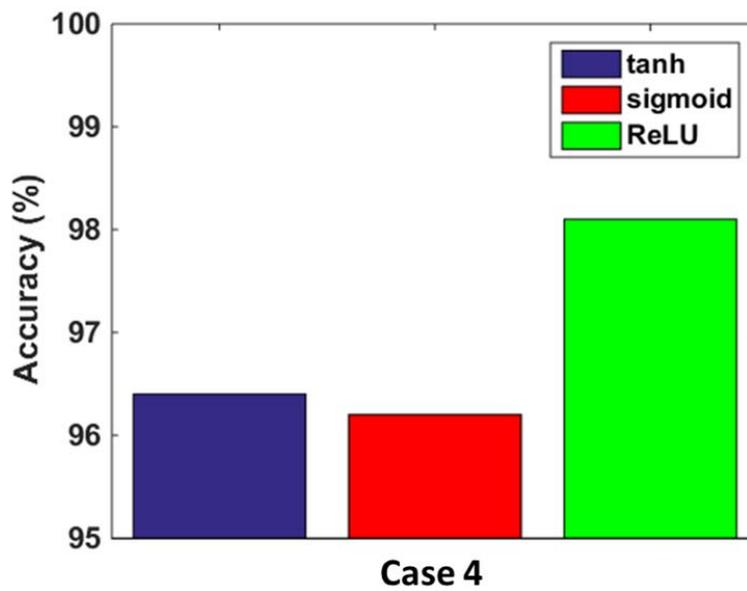**Figure 8 Comparison of the accuracy with different number of hidden layers for Case 4.**

Figure 9 Comparison of the performance of different activation functions for the hidden layers in Case 4.
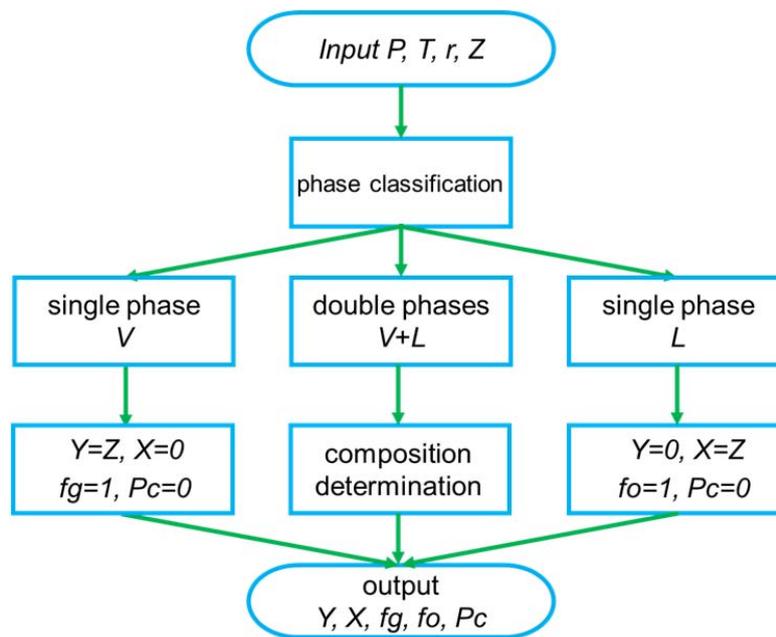
330 **4. Deep learning based flash calculator**

331   The trained deep learning (DL) network is implemented in our flash calculation module. The
332 proxy flash calculator provides the initial guess of the equilibrium ratio as well as the capillary pressure,
333 replacing Wilson's equation. Since the phase classification step is of very high accuracy, the phase
334 condition predicted by that step is adopted as the 'final' result. Therefore, if the phase classification step
335 predicts that only one phase exists in the system, the DL proxy calculator directly outputs the results. If,
336 however, the phase classification step predicts that two phases exist in the system, the flash calculator
337 will start the K-value based iteration using the predicted equilibrium ratio and capillary pressure, until
338 global convergence. A simplified and a detailed flowchart of the deep leaning based flash calculator
339 are shown in Figure 10 and Figure 11 respectively.

340   The comparison between the average number of iterations of the flash calculation with and
341 without DL preconditioning is shown in Table 3. According to the comparison of the numerical
342 performance, the number of iterations has been cut by above 50%. According to the results, DL based
343 proxy calculation effectively reduces the number of iterations of flash calculation and, thus accelerates
344 the reservoir simulation.

345   Moreover, we have observed that DL based preconditioner effectively improves the stability
346 (convergence) of flash calculation. The large capillary effect causes the flash calculation with Wilson's
347 initial guess to be difficult to get converged, which prohibits its applications. As shown in Table 4, with
348 the implementation of the DL based preconditioner, the ratio of the converged flash calculations among
349 the 300,000 data sets (parameters shown in Table 1) increases from 90% to above 98%. This is also
350 because that the DL based preconditioner provides a much more accurate initial guess to the flash
351 calculation, making it close enough to the real solution for the Newton-based algorithm to converge.
352

Figure 10 Flowchart of the proxy flash calculation in a reservoir simulator.

Given z, r, P, T

Initialization by DL proxy calculator

Single phase?

Yes

No

Rachford-Rice equation

Fugacity by PR-EOS with capillary pressure

$$\sigma_{VL} = \left(P_L \rho_L - P_V \rho_V \right)^v$$

$$P_c = p^V - p^L = \frac{2\sigma cos\theta}{r}$$

$$K_i = \frac{\Phi_i^L p^L}{\Phi_i^V \left(p^L + P_c \right)}$$

Fugacity criteria

update K_i

convergent

Output solution

Figure 11 Flowchart of the deep leaning based flash calculator.

Table 3 Comparison of the number of iterations with and without the number of deep-learning based preconditioner.

Case index	Feed component	Iterations without DL preconditioner	Iterations with DL preconditioner
1	C1+C2+C3	3.7	1.3
2	C1+C2+C3+C6	8.6	2.0
3	C1+C2+C3+C4+C5	14.3	2.2

| 4 | C1+C2+C3+C5+C7+C9 | 17.0 | 3.6 |
| 5 | C1+C2+C3+C4+C5 C8+C9+C10 | 25.8 | 4.5 |

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Table 4 Comparison of the ratio of convergence with and without the number of deep-learning based preconditioner.

Case index	Feed component	Ratio of convergence without DL preconditioner	Ratio of convergence with DL preconditioner
1	C1+C2+C3	96.1%	99.4%
2	C1+C2+C3+C6	94.9%	99.1%
3	C1+C2+C3+C4+C5	93.4%	98.8%
4	C1+C2+C3+C5+C7+C9	92.7%	98.5%
5	C1+C2+C3+C4+C5 C8+C9+C10	91.3%	98.3%

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372

373 **5. Case study**

374   We have implemented the deep-learning based flash calculator into our simulator
375 MSFLOW_CO2 and have conducted several case studies to investigate the performance of the deep
376 learning based compositional simulator. In this chapter, all numerical cases are executed by an Intel i7-
377 6700 processor with 3.40 GHz. We have investigated the compositional simulation of a fractured
378 reservoir. The reservoir is naturally fractured. A horizontal well is drilled through the reservoir, and a
379 hydraulic fracture is engineered within the reservoir, creating a stimulated reservoir volume (SRV) in the
380 vicinity of the hydraulic fracture. The conceptual model of the problem is shown in Figure 12, in which
381 the green and the orange part indicates the area within and outside the SRV, respectively.

382   The natural fractured reservoir part and the SRV are both modeled as dual-porosity systems. In
383 the dual-porosity system, the fracture network provides flow channel while the matrix rock stores the
384 hydrocarbon. A 'shape-factor' [46–48] is used to quantify the flow between the matrix rock and the
385 fracture network. Wu and Pruess [49] incorporated the shape factor into the integrated finite difference
386 (IFD) framework. Therefore, in this work the single-continuum and dual-porosity model are both
387 discretized using the same IFD approach, as suggested by Wu and Qin [50]. The length of the entire
388 reservoir along *x*- and *y*- direction is 540m and the 400m, respectively, while the length of the SRV
389 along *x*- and *y*-direction is 120m and 160m, respectively. The conceptual model of the case is shown in
390 Figure 12. The relative permeability of the gas phase and the oil phase is modeled by the classic
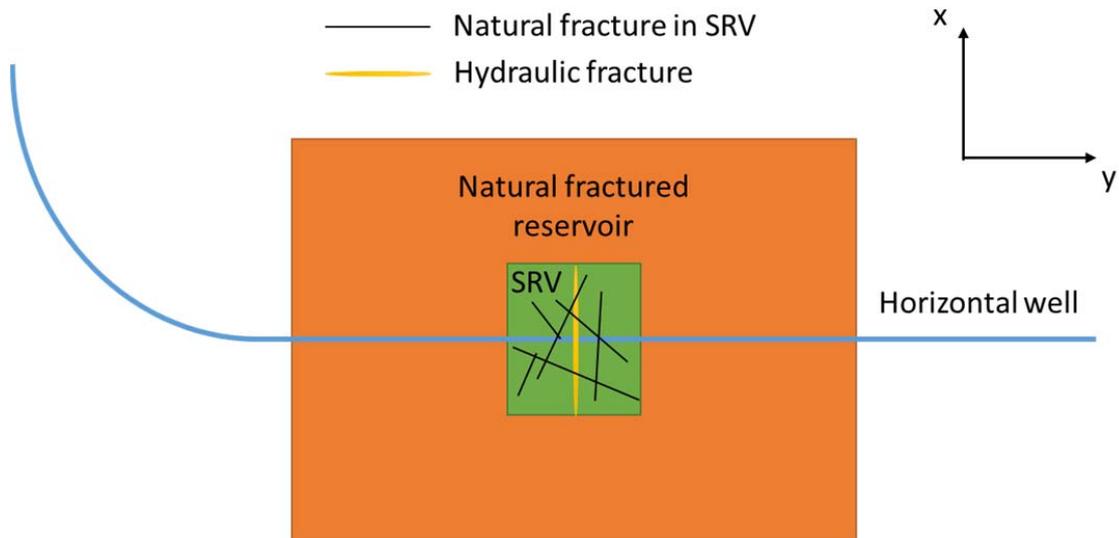
391 Brooks-Corey model [51], as shown in Equation 34 and 35, where S_{or} and S_{gr} refers to the residual

392 saturation of the oil and gas phase, respectively. $k_{ro,max}$ and $k_{rg,max}$ refer to the maximum relative

393 permeability of the oil and gas phase, respectively. n_o and n_g are two constants.

$$394 \quad k_{ro} = k_{ro,max} \left(\frac{S_o - S_{or}}{1 - S_{or} - S_{gr}} \right)^{n_o} \qquad\qquad 34$$

$$395 \quad k_{rg} = k_{rg,max} \left(\frac{S_g - S_{gr}}{1 - S_{or} - S_{gr}} \right)^{n_g} \qquad\qquad 35$$

396   The initial distribution of components of this case is shown in Table 5. The geomechanical

397 impact is not considered in this case. The input parameters, including the rock properties, are listed in

398 Table 6. We run this case with three different grid block sizes, namely 10 m*10 m, 8 m*8 m and 4 m*4

399 m for 8 years. As the results, the oil pressure fields of the fracture system and the matrix rock at the

400 end of the production are shown in Figure 13. The comparison of the oil saturation fields of the matrix

401 rock system at the end of the first year of production and at the end of the eighth year of production is

402 shown in Figure 14. According to the results, the fractures in the vicinity of the hydraulic fracture get

403 quickly drained by the production, while the pressure inside the matrix rock system declines much

404 slower. As the pressure decreases, the gas phase expands and the oil saturation decreases.

405   We compare three types of initialization strategy of the flash calculation, namely initializing by

406 Wilson's equation, initializing by the primary variable from the previous NR iteration step, and initializing

407 by the DL preconditioner. In the second type, the initial guess of the equilibrium ration of the flash

408 calculation is calculated by the primary variable of the last Newton-Raphson iteration step of the same

409 grid block. In the second, the equilibrium ratio is obtained from the deep learning based proxy flash

410 calculation module. The comparison of the CPU time of the three types with different grid block sizes is

411 shown in Figure 15. According to Figure 15, the DL preconditioner effectively reduces the CPU time by

412 about 10% to 12%, compared to initializing by the equilibrium ratio from the previous iteration step. The

413 results presented in this case show that DL preconditioner is capable of accelerating the performance

414 of large scale compositional simulation for unconventional reservoirs.

Figure 12 Conceptual model of the case study with the DL based reservoir simulator.

Table 5 Initial distribution of components for deep learning based compositional reservoir simulation case.

Component	Formula	Mole fraction
Methane	CH_4	0.40
Ethane	C_2H_6	0.15
Propane	C_3H_8	0.15
Butane	C_4H_{10}	0.05
n-Pentane	C_5H_{12}	0.05
n-Heptane	C_7H_{16}	0.20

Table 6 Input parameters for fractured reservoir case.

Property	Value	Unit
Permeability of the matrix rock	0.1	µd
Porosity of the matrix rock	0.01	dimensionless
Permeability of the hydraulic fracture	100	md
Porosity of the hydraulic fracture	0.2	dimensionless
Permeability of the fractures in SRV	50	md
Porosity of the fractures in SRV	0.1	dimensionless
Permeability of the fractures outside SRV	20	md

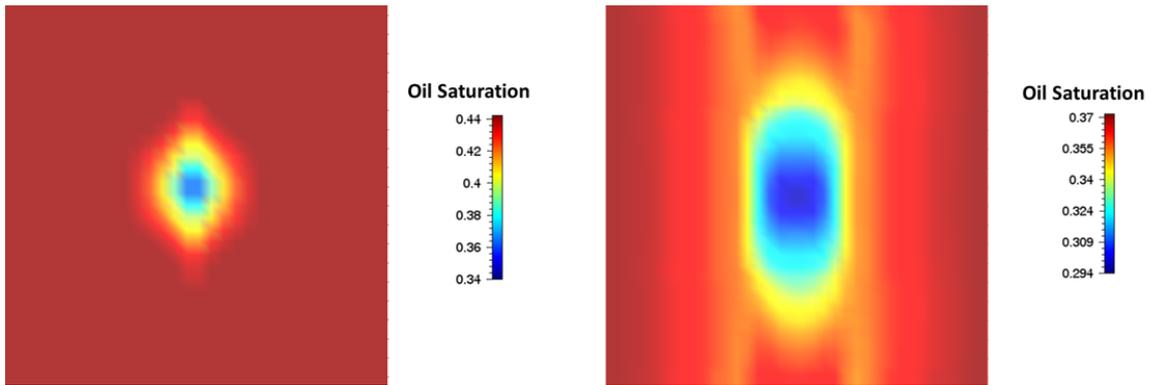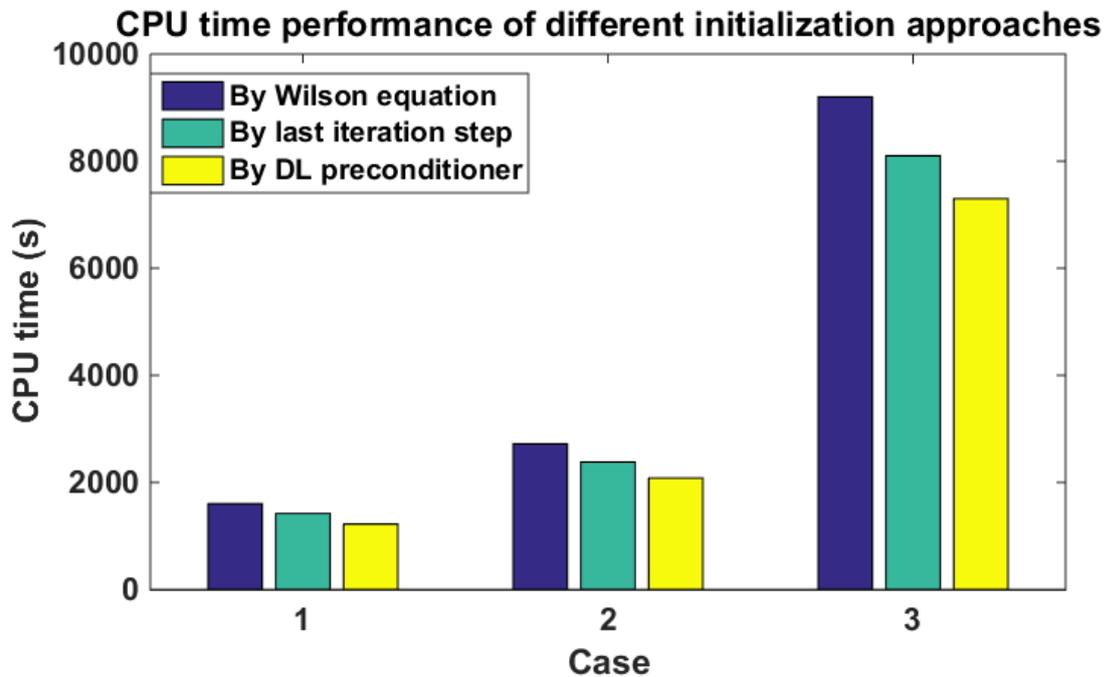|		
Porosity of the fractures outside SRV	0.05	dimensionless
Rock compressibility	0.0	dimensionless
Initial pressure	21.2	MPa
Initial temperature	120	°C
Production pressure	10.2	MPa
Pore radius	50	nm
Residual gas saturation (S_{gr})	0.1	dimensionless
Residual oil saturation (S_{or})	0.1	dimensionless
Maximum gas relative permeability ($k_{rg,max}$)	0.7	dimensionless
Maximum gas relative permeability ($k_{ro,max}$)	0.9	dimensionless
n_g	2.0	dimensionless
n_o	2.0	dimensionless

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425
426   **Figure 13 Comparison of the oil pressure fields of the matrix rock system and the fracture**
427   **system at the end of the 8-years production. Left: matrix rock system. Right: fracture system.**
428
429
430

Figure 14 Comparison of the oil saturation fields of the matrix rock system at the end of the first year of production and at the end of the eighth year of production. Left: end of the first year. Right: end of the eighth year.

Figure 15 Comparison of the CPU time of the compositional reservoir simulation cases initialized with different initial guesses.

6. Summary and Conclusion

To sum up, we have developed a data-based proxy flash calculator to speed up the time-consuming flash calculation. The proxy flash calculator adopts an initial guess obtained from the deep neural network, the accuracy of which is above 95%. With the implementation of the proxy calculator, the number of iterations of the flash calculation has been effectively reduced by about 50%. Moreover, the stability of the flash calculation has been improved by the DL based preconditioner, with the ratio of convergence increased from 90% to above 98% percent. This work is among the first trials in this area.

449   In the present work, the fully-connected neural network is used for the proxy flash calculator. In
450 the future, other structures can also be tried and compared, for instance, the convolutional neural
451 network and the recurrent neural network. Moreover, other portions of the simulator may also be
452 accelerated by the deep learning techniques. For example, the wellbore flow part, which is a very time-
453 consuming simulation of multiphase flow, can also be replaced by a DL based proxy calculator.

454

455 **Acknowledgments**

457

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581   **Appendix A Parameters for flash calculation**

582
583   Table A.1 Parameters for the calculation of component properties

	T_c (K)	P_c (MPa)	w	M_w ($g \cdot mol^{-1}$)	a_0 ($KJ \cdot (kg \cdot K)^{-1}$)	a_1 ($KJ \cdot (kg \cdot K)^{-1}$)
CH_4	190.56	4.599	0.0115	16.043	2.191	0.002672
C_2H_6	305.32	4.872	0.0995	30.07	1.651	0.004384
C_3H_8	369.83	4.248	0.1523	44.096	0.79	0.00468
$n\text{-}C_4H_{10}$	408.14	3.648	0.2002	58.123	0.818	0.004255
$n\text{-}C_5H_{12}$	469.7	3.37	0.2515	72.15	-0.218	0.001895

n-C$_6$H$_{14}$	507.6	3.025	0.3013	86.177	-0.491	0.007187
n-C$_7$H$_{16}$	540.2	2.74	0.3495	100.204	-0.756	0.007811
n-C$_8$H$_{18}$	568.7	2.49	0.3996	114.231	-0.989	0.00836
n-C$_9$H$_{20}$	594.6	2.29	0.4435	128.258	-1.236	0.008951
n-C$_{10}$H$_{22}$	617.7	2.11	0.4923	142.285	-1.465	0.009484
CO$_2$	304.3	7.39	0.2236	44.01	0.727	0.003722

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Table A. 2 Parachor values for all hydrocarbon components

Component	Parachor
CH$_4$	77.0
C$_2$H$_6$	108.0
C$_3$H$_8$	150.3
n-C$_4$H$_{10}$	203.4
n-C$_5$H$_{12}$	231.5
n-C$_6$H$_{14}$	271.0
n-C$_7$H$_{16}$	312.5
n-C$_8$H$_{18}$	351.5
n-C$_9$H$_{20}$	393.0
n-C$_{10}$H$_{22}$	617.7

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