3D Traveltime Reflection Tomography with Multi-valued Arrivals

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Summary

We present a new formulation of 3D traveltime reflection tomography designed to invert multi-valued arrivals. It is applied on a 3D data set recorded over a salt ridge from the North Sea.

Introduction

Classical reflection tomography generally furnishes a velocity model which satisfies data with a single traveltime per source-receiver pair. However, when dealing with complex structures there are often several traveltimes for a given source-receiver pair (multi-pathing). This poses a basic difficulty for reflection tomography : the forward modelling operator is then a multivocal operator. In practice, this leads to the following difficulty : as the velocity model changes, the number of arrivals for a given source-receiver pair may vary from one iteration to the next and, as it is difficult to know which traveltime and observations should be compared, convergence problems are encountered.

The most common solution to this problem involves systematically using the first arrival time during the inversion. However, areas containing multivalued traveltimes are then poorly determined. Consider how a simple syncline may produce a classical triplication. Inverting just the first arrival times, the reflector is only partly illuminated, and, as a result, it will be poorly determined. Of course, this horizon can also create triplications for deeper reflectors as the rays transmit across it. It should also be remembered that the velocity distribution can also create triplications (e.g. velocity inclusions such as salt bodies, over pressured zones, gas chimneys,...).

Methodology

Delprat-Jannaud and Lailly (1995), proposed a formulation which makes use of the ray parameters at the source to parameterize the data and 'unfold' multi-valued kinematics. They look for a model such that, when shooting rays in this model from a given source location with specified ray parameters at the source, rays emerge according to observed data (receiver location and traveltime). The forward modelling operator is then definite: for a given source location and ray parameters at the source, they trace a ray and obtain a single emergence location and a single traveltime.

Delprat-Jannaud (1995), successfully illustrated the efficiency of this method in 2D, but highlighted a difficulty for the extension to 3D: the method requires precise knowledge of the ray parameters, which are related to the slope of the traveltime surface. But, typical 3D seismic marine acquisition prevents us from accurately estimating the ray parameter in the crossline direction, due to the large source-receiver spacing in this direction.

The zero offset data (from a near offset stack), on the other hand, provides a fine source-receiver spacing in both directions. Thus, both the ray parameters can be obtained accurately, either by finite difference, or by demigrating depth horizons ((Ehinger and Lailly, 1995)). Hence, this formulation of reflection tomography is currently only feasible in 3D for zero offset data, and another approach is required for multi-offset data, which is necessary for reflection tomography to function correctly.

New formulation of the inverse problem

We propose to use the formulation of Delprat-Jannaud (1995) for zero offset data only, where the ray parameters may be precisely obtained, and to associate the different multi-offset traveltime branches with their zero offset rays in order to handle multi-offset data. In this way, the multi-offset data may be multi-valued if the corresponding zero offset data is multi-valued (see Figure 1).

For the zero offset data, rays are shot from a source with the correct ray parameters and, because the current velocity model is usually incorrect, do not emerge at the receivers. The cost function to be minimized is

$$\underbrace{\sum_{i=1}^{N_{zorays}} \begin{pmatrix} \Delta t_i \\ \Delta x_i \\ \Delta y_i \end{pmatrix}^t C_i^{-1} \begin{pmatrix} \Delta t_i \\ \Delta x_i \\ \Delta y_i \end{pmatrix}}_{\text{zero-offset data}} \qquad (1)$$

where N_{rays} is the number of rays traced, Δt_i is the misfit in the traveltime, Δx_i and Δy_i are the misfit in the (x, y) co-ordinates of the emergence point, and C_i^{-1} is a covariance matrix of data uncertainties:

$$C_i^{-1} = \begin{pmatrix} < \delta t_i^2 > & < \delta t_i \delta x_i > & < \delta t_i \delta y_i > \\ < \delta x_i \delta t_i > & < \delta x_i^2 > & < \delta x_i \delta y_i > \\ < \delta y_i \delta t_i > & < \delta y_i \delta x_i > & < \delta y_i^2 > \end{pmatrix}$$

The covariance matrix is essential for several reasons:

- The objective function uses several different units (e.g. seconds for Δt_i, metres for Δx_i) and the magnitudes of the three misfit terms can be different.
- The misfits in the traveltime data and the emergence location data are correlated.

- Also, a few rays can be extra sensitive to certain model perturbations and their contribution to the Hessian of the objective function leads to a large condition number, and subsequently very poor convergence (Delprat-Jannaud and Lailly, 1993b).

When obtaining the zero offset data through demigration, uncertainties in the picked horizon are translated into traveltime and emergence location uncertainties (Clarke, 1997), which are the elements of the covariance matrix.

Of course, there are cases where the ray may fail to emerge. For example, if there are strong velocity contrasts at interfaces, then rays may fail to transmit, or rays may exit by the sides of the model. Also, the vertical component, p_{z_s}, of the slowness vector at the source is calculated from $p_{x_s}^2 + p_{y_s}^2 + p_{z_s}^2 = u_s^2$, the condition $p_{x_s}^2 + p_{y_s}^2 \leq u_s^2$ must be satisfied. But, apart from these pathological cases, the forward map is always defined.

For the multi-offset data we use a classical formulation and the cost function to be minimized is

$$\underbrace{\sum_{i=1}^{N_{branches}} \left(\sum_{j=1}^{N^i_{morays}} \frac{(t_j^{calc} - t_j^{tobs})^2}{\sigma_j^2} \right)}_{\text{multi-offset data}} \qquad (2)$$

where $N_{branches}$ is the number of multi-offset traveltime branches ($N_{branches} \leq N_{zorays}$), N^i_{morays} is the number of multi-offset rays on branch i, t_j^{calc} is the (two point) multi-offset traveltime calculated by raytracing, t_j^{obs} is the observed traveltime, and σ_j^2 is an uncertainty term, similar to the diagonal terms $< \delta t_i^2 >$ of the covariance matrix, C_i^{-1}.

Minimizing (1) and (2) together (with some weights to balance the two terms) should yield a velocity model which satisfies the data. In other words, rays traced in the solution model will emerge at the receivers with the correct traveltime even in the case of complicated kinematics (multivalued traveltimes).

Further penalization terms in the objective function for regularizing the model (Delprat-Jannaud and Lailly, 1993a), as well as equality and inequality constraints (Sinoquet, 1995), and (Jurado et al., 1996) allow a priori information to be added. These constraints are handled using an augmented Lagrangian technique (Glowinski and Tran, 1993).

Forward Modelling

It is worth noting that the forward modeller of classical tomography is usually two point raytracing, which is computationally costly, particularly for 3D complex structures. By making use of the initial shooting directions of the rays, the new formulation replaces the two point raytracing by a simple shooting, which is a trivial problem and much less computationally expensive but it requires to know the ray parameters at the source. Jacobian calculation for the inversion is done by use of an adjoint state technique (Lee and Markus, 1967; Bécache, 1992).

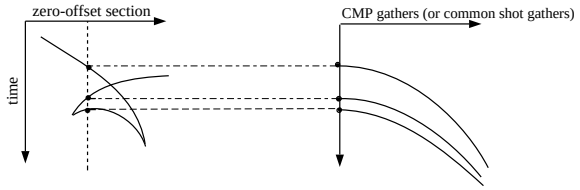

Fig. 1: Triplications are possible for the zero offset data. These are then continued into the multi-offset data.

A constrained offset continuation raytracing technique is used to ensure that the forward modeller always obtains traveltimes corresponding to the same traveltime branch as the zero offset ray. The *zero offset* ray is shot (and does not necessarily emerge at the receiver) and is used to initialise the continuation technique. The initial conditions of the ray are perturbed, using the derivative information from the paraxial matrix, in order to find a ray which emerges closer to the new receiver location. In other words, the distance between the emergence point of the ray and the receiver location is minimized. At each iteration the convergence and the KMAH index (the number of caustics crossed by the ray) is checked in order to ensure that only rays belonging to the same traveltime branch as the zero offset ray are calculated. Hence, there is no ambiguity in the modeling operator: for each observed traveltime there will be at most one calculated traveltime.

The picking and the forward modelling strategies are illustrated schematically in Figure 1. For the picking, we obtain triplicated zero offset data through depth domain picking and demigration, followed by picking the multi-offset data using the zero offset times as a guide. For the forward modelling, we model triplicated zero offset data by shooting rays with given source locations and ray parameters, then we use these rays to initialize a continuation procedure for the two point multi-offset rays.

Examples

We apply the method on a 3D North Sea dataset (Figures 2 and 3) covering an area approximately $13km$ by $11km$. The subsurface structure is a salt ridge with two piercing salt domes. The data were acquired in the dip direction with two sources and two streamers, with an in-line shot spacing of $25m$, an in-line receiver spacing of $26.66m$, and a cross-line spacing of $25m$.

We concentrate here on the inversion of the Tertiary velocity and base reflector, which generate multi-valued traveltimes. We assume the vertical velocity gradient to be constant at $k = 0.5^{-1}$. As initial model we use the model obtained by (Broto and Ehinger, 1998), who applied classical tomography using first arrival traveltimes

from a coarse grid of CMPs and from 17 regularly spaced acquisition lines.

We flood the model below the Tertiary layer with the Tertiary velocity and run post-stack depth migration. Interpretation is performed in the depth domain as presented in Figure 3 (Clarke, 1997). According to the confidence we have in the depth interpretation, we choose an RMS uncertainty on the picked horizon of $10m$ and a correlation length of $1km$, from which the raytracer also calculates the traveltime and emergence point uncertainties to fill the covariance matrix. To obtain the multi-offset traveltimes, we use the zero offset times as a guide and pick in the common shot domain (Figure 4. In turns out that the CMP domain would have been preferable for several reasons: it is easier to pick, and the smearing of the impact points with the offset is smaller, which makes the continuation technique more stable. However, by demigrating some near offset times[1] and using them as a guide also, we succeeded in obtaining a total of approx. 65000 traveltimes, of which 2800 were 'secondary' arrivals.

Tomographic inversion converges to a satisfactory model after about 20 iterations. We observe that this model explains the observed data quite well. The RMS misfit for multi-offset traveltimes is $17ms$, the RMS misfit for the zero offset times is $5ms$, and the RMS misfits for the zero offset x and y emergence locations are $23m$ and $15m$. In particular, we now observe a much improved consistency between calculated and observed data in the zone of triplications (see Figure 8) compared to the results for the initial model (see Figure 7).

Figures 5 and 6 display a comparison between the initial velocity model and the obtained final model. The shape of the base of Tertiary layer has evolved quite significantly. Most noticeable, synclines have appeared thanks to the handling of multi-valued traveltimes.

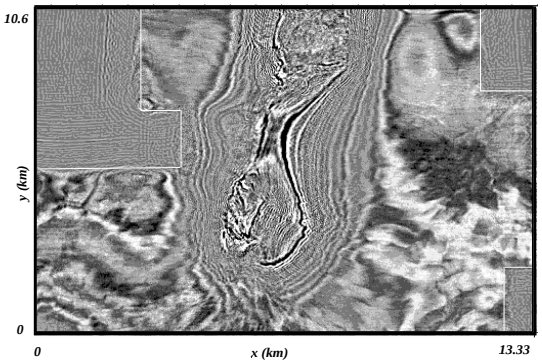

Fig. 2: Depth slice at $1040m$ through the salt body. The zones delimited in white contain no seismic data.

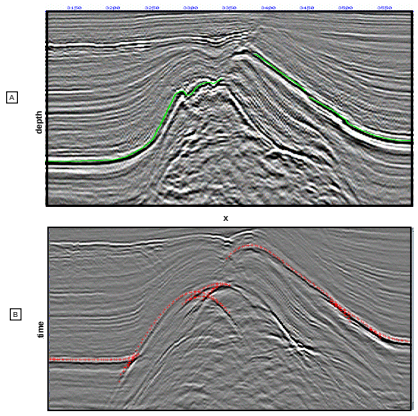

Fig. 3: Inline section 1414 of the stack cube. Steps 2 and 4 of the SMART zero-offset method: depth picking (A) et demigration (B).

Conclusions

We have proposed a new 3D tomography formulation designed for inverting the multi-valued traveltimes which are generated by complex geological structures. The formulation overcomes the problem of sparse crossline sampling which is typical of 3D marine surveys by using the ray parameters from stacked (zero offset) data to unwravel triplications, combined with a continuation technique for multi-offset data. The method has been succesfully applied on a 3D salt dome example from the North Sea.

Acknowledgements

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[1] These traveltimes will strictly speaking only be precise for zero offset, as we use the post-stack depth migrated volume and demigrate with the migration velocity. They are used *only* as an interpretation guide.

References

Bécache, E., 1992, Reflection tomography: how to cope with multiple arrivals? Part II: A new gradient computation method: continuous and discrete problems: PSI Annual Report, Institut Français du Pétrole, Rueil-Malmaison, France.

Broto, K., and Ehinger, A., 1998, How to access 3D prestack kinematic information in case of complex ge-

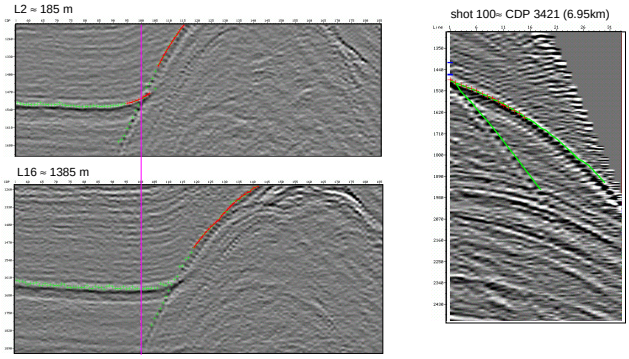

Fig. 4: Multi-offset branches interpreted by continuation, first and second branch correspond to flat and sloping parts of the syncline, respectively.

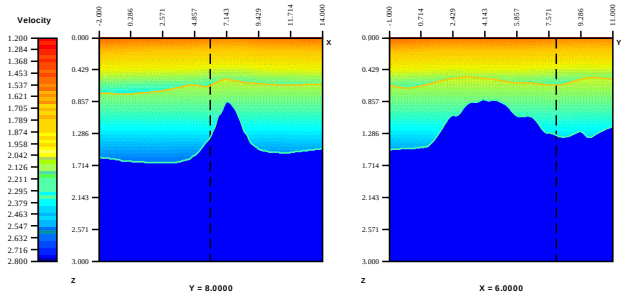

Fig. 5: The initial velocity model obtained from first arrival traveltime tomography.

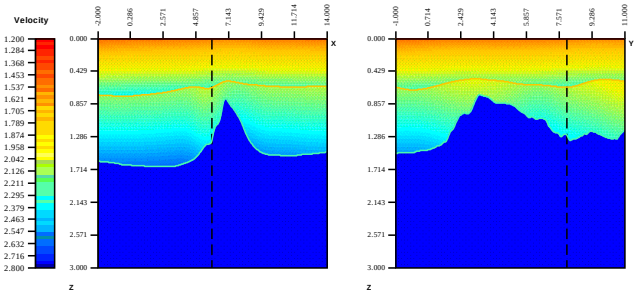

Fig. 6: The final velocity model obtained with our new traveltime tomography.

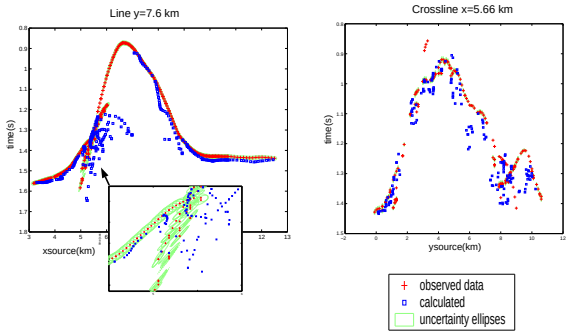

Fig. 7: Traveltime misfits and uncertainties with the initial velocity model obtained from first arrival traveltime tomography.

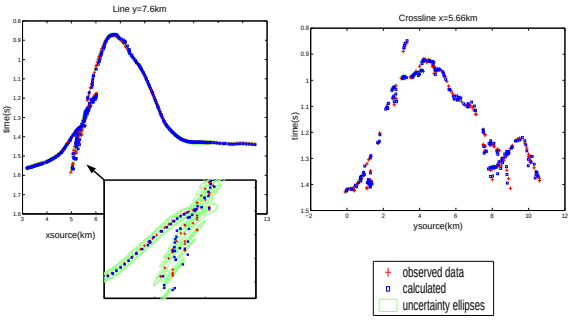

Fig. 8: Traveltime misfits and uncertainties with the final velocity model obtained with our new traveltime tomography.

ologies?: 68^{th} Ann. Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts, **2**, 1401–1404.

Clarke, R. A., 1997, Modélisation et inversion de données cinématiques complexes en 3D (in English): Ph.D. thesis, Université de Pau et des Pays de l'Adour, France.

Delprat-Jannaud, F., and Lailly, P., 1993a, Ill-posed and well-posed formulations of the reflection travel time tomography problem: Journal of Geophysical Research, **98**, 6589–6605.

———– 1993b, Reflection tomography with multiple arrivals: How to handle noise corrupted data?: 1993 PSI Annual Report, Institut Français du Pétrole, Pau, France, pages 267–298.

Delprat-Jannaud, F., and Lailly, P., 1995, Reflection tomography: how to handle multiple arrivals?: Journal of Geophysical Research, **100**, no. B1, 703–715.

Delprat-Jannaud, F., 1995, Sensitivity to the inversion algorithm to an error in the ray parameter: 1995 PSI Annual Report, Institut Français du Pétrole, Pau, France, pages 81–102.

Ehinger, A., and Lailly, P., 1995, Velocity model determination by the SMART method, Part 1: Theory: 65^{th} Ann. Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts, pages 739–742.

Glowinski, R., and Tran, Q., 1993, Constrained optimization in reflection tomography: the Augmented Lagrangian method: East-West J. Numer. Math., **1**, 213–234.

Jurado, F., Sinoquet, D., and Ehinger, A., 1996, 3D reflection tomography designed for complex structures: 66^{th} Ann. Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts, 711–714.

Lee, E. B., and Markus, L., 1967, Foundations of optimal control theory: John Wiley and Sons Inc.

Sinoquet, D., 1995, Utilisation de modèles lisses pour l'inversion tomographique de données sismiques (in English): Ph.D. thesis, Université Paris XIII, France.