A dedicated constrained optimization method for 3D reflexion tomography

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Abstract

Seismic reflection tomography is a method for the determination of a subsurface velocity model from the traveltimes of seismic waves reflecting on geological interfaces. From an optimization viewpoint, the problem consists in minimizing a nonlinear least-squares function measuring the mismatch between observed traveltimes and those calculated by raytracing in this model. The introduction of a priori information on the model is crucial to reduce the under-determination. The contribution of this paper is to introduce a technique able to take into account geological a priori information in the reflection tomography problem expressed as constraints in the optimization problem. This constrained optimization is based on a Gauss-Newton Sequential Quadratic Programming approach. At each Gauss-Newton step, a solution to a strictly convex quadratic optimization problem subject to linear constraints is computed thanks to an augmented Lagrangian relaxation method. Our choice for this optimization method is motivated and its original aspects are described. The efficiency of the method is shown on applications on a 2D OBC real data set and on a 3D real data set: the introduction of constraints coming both from well logs and from geological knowledge allows to reduce the under-determination of the 2 inverse problems.

Introduction

Reflection tomography allows to determine a velocity model from the traveltimes of seismic waves reflecting on geological interfaces. This inverse problem is formulated as a nonlinear least-squares function which measures the mismatch between observed traveltimes and traveltimes computed by ray tracing method. This method has been successfully applied to real data sets (Ehinger et al, 2001, Broto et al, 2003). Nevertheless, the under-determination of the inverse problem generally requires the introduction of additional information on the model to reduce the number of admissible models. Penalization terms modelling this information can be added to the seismic terms in the objective functions but the tuning of the penalization weights may be tricky. In this paper, we propose to handle the a priori information by the introduction of equality and inequality constraints in the optimization process. This approach allows to introduce lot of constraints of different types, provided we have at our disposal an adequate constrained optimization method. We developed an original method designed for the tomographic inverse problem which presents the following characteristics: it is a large scale problem (10000-50000 unknowns), the forward operator is nonlinear and its computation may be expensive (large number of source-receiver couples, up to 500000), the problem is ill-conditioned.

In the first part of this paper, the chosen method is motivated and its original aspects are shortly described (for further details, refer to Delbos et al, 2004). Applications on a 2D PP/PS real data set and on a 3D PP real data set are presented in a second part.

Constrained optimization method designed for reflection tomography

The general problem in constrained reflection tomography can be formulated as

$$Minimize \ \left(f(m) := \frac{1}{2}\|T(m)-T_{obs}\|_2^2 + \frac{\sigma^2}{2} m^T Rm \right) \text{ subject to } (C_E m = e) \text{ and } (l \le C_I m \le u),$$

where f is the objective function, m the parameters of the model, T is the nonlinear traveltime operator, R is the regularization matrix, σ is the associated regularization weight and C are the equality or inequality constraint matrices. In this last formulation, we recognize the standard regularized nonlinear least-squares problem of reflection tomography subject to linear constraints. Two main approaches can be used to solve this nonlinear optimization problem: the penalty methods including the famous Interior Points approach (IP) and the Sequential Quadratic Programming approach (SQP). The current stage of the IP method seems to require more cost function evaluation than the SQP does. This is our motivation to develop an SQP approach for solving the reflection tomography problems considering that the ray tracing solver is expensive in CPU time.

The SQP method consists in solving a sequence of quadratic problems subject to linear constraints. To be efficient the SQP method must solve each quadratic problem, called also the Tangential Quadratic Problem (TQP), in a reasonable CPU time. The TQP to be solved at the Gauss-Newton step k can be formulated as

$$(\mathrm{TQP})_k : \quad \textit{Minimize } F_k(\delta m) \text{ subject to } (C_E.\delta m = e_k) \text{ and } (l_k \leq C_I.\delta m \leq u_k),$$

where F_k is the quadratic Gauss-Newtonian approximation of the nonlinear function f in m_k and δm is the model perturbation to be computed from m_k .

Three main approaches can be used to solve this convex quadratic problem subject to linear constraints : Interior-Points approach (IP), Augmented Lagrangian approach (AL) and Active Set approach (AS). AS approach has been discarded because of its inefficiency to quickly identify the active constraints. In the present context the potential ill-conditioning of the linear system associated with IP method and the necessity to use iterative methods to solve them appear to us as a deterrent factor. Then, we believe that an AL method implemented in such a way that it does not require any matrix factorization is the appropriated method to solve the TQP.

The AL method consists in solving a sequence of quadratic problems subject to bound constraints. Each of them, called the Lagrange Problem (LP), is efficiently solved by an original method combining the three following ingredients: the projected gradient method, the active set algorithm and the conjugate gradient iterations. At each AL iteration, the Lagrange multipliers are updated thanks to the classical multiplier method of Hesteness and Powell.

The overall method explained above has been successfully tested on several reflection tomography applications (the next section presents two of them). For further details on the different optimization method used to solve our constrained reflection tomography problem, we refer the reader to Delbos (2004).

Applications of constrained reflection tomography on real data sets

2D PP/PS data set: In this section, we present an application of constrained reflection tomography to one 2D line of a 3D 4C OBC survey with PP and PS data. Broto et al. (2001) have already interpreted and studied this data set using an unconstrained inversion method. The velocity model is described by four velocity layers and five interfaces (cf Figure 1 left). The isotropic assumption was satisfying until the last layer (layer which contains the last two interfaces h4 and h5). By applying the anisotropic inversion methodology of Stopin (2001) on the last layer, they obtained a model that fits the traveltimes better than any of the previous isotropic models and that, in addition, has more reliable velocity variations. The value of δ anisotropy parameter has been obtained by a trial and error approach in order to match approximately the h5 depth given by well logs. Actually, the under-determination of the inverse problem does not allow the joint inversion of the velocities, interfaces and anisotropy parameters (especially δ parameter, Stopin, 2001).

This applied methodology is obviously not optimal. Indeed, the manual tuning of the anisotropy parameters requires lot of time: an important numbers of anisotropic inversion with different couples (η,δ) have to be performed before getting a satisfying result. Secondly, it is very hard to make this method accurate: we note a discrepancy of 150 meters for the reflector depth h5 compare to the depth given by the well data. Finally, it turned out impossible to determine the anisotropy parameter δ so that both the reflector depths of h4 and h5 given by the well logs be reasonably matched.

The solution we propose here is to compute a model using our constrained inversion method in order to fit with the reflector depths given by the well. In the left and right part of Table 1 we have respectively summed up the results of the final models obtained with the unconstrained inversion and with a constrained inversion. The final model (Figure 1 right) of the constrained inversion matches the traveltime data with the same accuracy than the result obtained by unconstrained inversion[1], and it strictly verifies the reflector depths given by well logs.

Then, the introduction of constraints at wells for the two reflectors h4 and h5 reduces the under-determination and allows a joint inversion of velocities, interfaces and anisotropy parameters to find a

[1] The smooth velocity model does not allow smaller traveltime misfits than 6.3ms when h4 depth is constrained at well location. Allowing a velocity discontinuity contrast at h4 interface may help to obtain smaller misfits.

model that matches the traveltime data and the well data.

I4	RMS (s)	Delta z (m)	η	δ
h4-PP	3,6	190	6,2%	2,0%
h5-PP	4,6	150		
h5-PS	9,8			

I4	RMS (s)	Delta z (m)	η	δ
h4-PP	6,3	**0**	8,2%	15,9%
h5-PP	3,9	**0**		
h5-PS	8,1			

Table 1: Inversion results of the unconstrained inversion (left) and the constrained inversion (right)

3D PP data set: During the European KIMASI project, reflection tomography was applied on a 3D North Sea data set from bp (Ehinger et al, 2001). A P-velocity model was obtained thanks to a top-down layer-stripping approach where lateral and vertical velocity variations within Tertiary, Paleocene and Cretaceous (divided in two velocity layers) units have been determined sequentially (Figure 2). A strong velocity under-determination in the upper Tertiary layer was detected during the inversion process due to the large layer thickness (2.5km) and to the very poor ray aperture. Several inversions with different velocity parameterizations gave the same traveltime misfit RMS. These different tests are time consuming (each test is a whole inversion) and the reflector depths given by well data are not well retrieved, this information being not explicitly introduced in the inversion process. To obtain a model more consistent with well data, we propose to apply our developed constrained tomographic inversion. The interface depths are constrained at 5 well locations and we constrain the range of variations of the vertical velocity gradient in the Tertiary layer thanks to well measurements (Table 2). To limit bad data fitting for deeper layers, a global inversion approach is applied. This global update is only possible if the layer thickness are constrained to avoid any non-physical intersection of interfaces. The experiment consists then in a global inversion of 127569 traveltimes for 5960 unknowns describing 4 velocity layers and 5 interfaces, subject to 2300 constraints[2]. The results are presented in Figure 3: the obtained model matches the data with the same accuracy as the model obtained by Ehinger et al (2001) and verifies all the introduced constraints. The total number of conjugate gradient iterations for each Gauss-Newton step[3] is less than 10000 (less than twice the number of unknowns), which is a very good result for a problem with 2300 constraints.

Constraints		**Model without constraints**	**Model with constraints**
Mean depth mismatch at well locations	tpal tchalk bchalk	96m 132m 140m	0m 0m 0m
Vertical velocity gradient in Tertiary	0.1 < k < 0.3	k=0	k~0.18/s
Velocity range	2.5 < vpal < 4 km/s 3.5 < vichalk < 5.7km/s 4.2 < vchalk < 5.8km/s	ok ok ok	ok ok ok

Table 2: Description of the equality and inequality constraints introduced in the inversion. Starting the inversion with the model obtained by unconstrained tomography (Ehinger et al, 2001), we succeed to obtain a model that matches the data and the 2300 constraints.

Conclusion

Reflection tomography often requires the introduction of additional a priori information on the model in order to reduce the under-determination of the inverse problem. A nonlinear optimization method that allows the introduction of linear constraints on the model has been developed. The applications on two real data sets have shown its efficiency.

References

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Acknowledgments

The authors would like to thank bp to have provided the data sets, Karine Broto and Anne Jardin for their precious comments on the applications and Carole Duffet for her crucial help.

[2] The constraints on the velocity variations and on the layer thickness are applied on a grid of 10x10x10 and 20x20 points.

[3] The total number of conjugate gradient iterations takes into account all the iterations of augmented Lagrangian method.

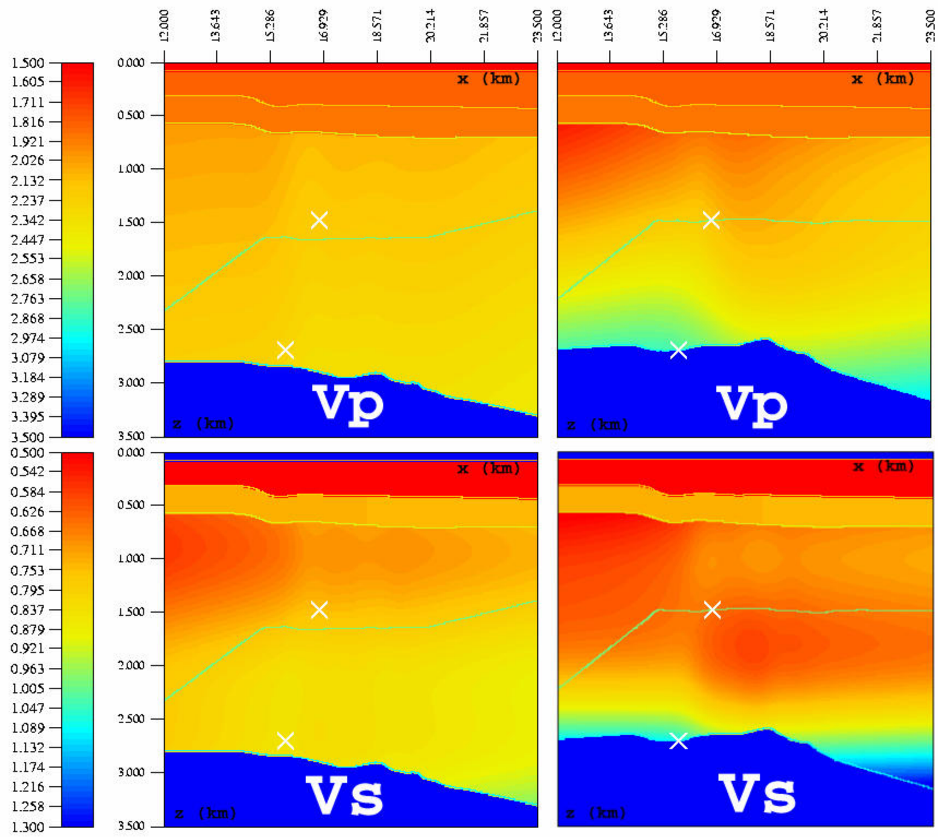

Figure 1: the left velocity models (Vp and Vs) are computed with the unconstrained inversion method; the right velocity models (Vp and Vs) are computed with the constrained inversion method. The white crosses symbolize reflector depths measured from the deviated well logs.

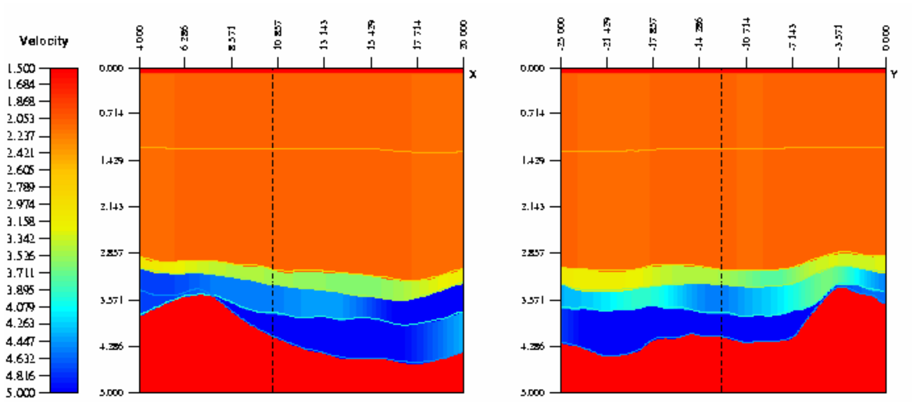

Figure 2: Velocity model (slices along x (left) and along y (right) at one of the 5 well locations) obtained with the unconstrained reflection tomography. The RMS value of the traveltime misfits is 6.1ms.

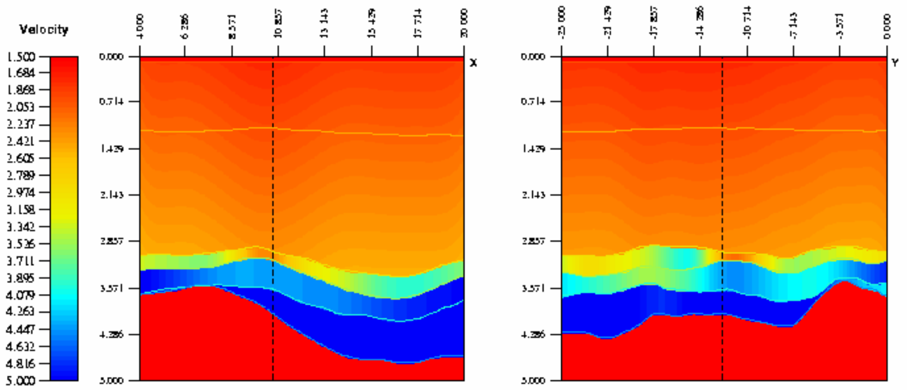

Figure 3: Velocity model (slices along x (left) and along y (right) at one of the 5 well locations) obtained with the constrained reflection tomography. The RMS value of the traveltime misfits is 6.5ms.