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Rate-of-return regulation to unlock natural gas pipeline deployment: insights from a Mozambican project ☆

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Abstract

In poor developing countries, the discovery of large gas deposits often stimulates the public authorities' appetite for ambitious development strategies requiring the construction of a large national pipeline system. However, the foreign private investors financing its installation usually prefer smaller infrastructure designs that are solely intended to supply a few creditworthy industrial sites. Focusing on the situation in Mozambique, we examine whether the adoption of rate-of-return (RoR) regulation can reconcile these conflicting objectives. As a first step, we assess the magnitude of the overcapitalization generated *ex ante* at the planning stage by the application of RoR regulation (i.e., the Averch-Johnson effect) to the investors. Then, analyzing the *ex post* situation when the enlarged domestic demand materializes, we prove that the allowable rate of return can be set by the regulator to obtain *ex ante* the degree of overcapitalization needed *ex post* to serve the enlarged demand in a cost-efficient manner. We finally discuss whether RoR regulation can still protect society from monopoly prices when it is tuned to prompt an optimal degree of building ahead of proven demand.

Keywords: Natural gas, Pipeline, Regulatory economics, Developing countries, Mozambique, Building-ahead of demand, Overcapitalization.

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1. Introduction

In developing nations, the discovery of large natural gas deposits is commonly depicted as a bonanza. However, translating that resource wealth into developmental achievements perceptible to the greater population is a challenging task with possibly dramatic consequences, as a failure can trigger regional tensions or even jeopardize the country's political stability (Caselli and Tesei, 2016). While pursuing this quest for inclusive development patterns, government planners are often tempted to leverage on such resource endowments to deliver measurable benefits to the population: an improved electrification rate supported by gas-fueled thermal power generation (Khennas, 2012), the production of fertilizers (e.g., urea, ammonia) to improve food security (Parikh et al., 2009), and the substitution of the low-quality and health-detrimental fuels used by households (Cesur et al., 2017). To become effective, this strategy necessitates the construction of a capital-intensive pipeline infrastructure and, because of the scarcity of domestic capital, needs to attract foreign direct investment (FDI) into a durable, specific asset.

In least developed countries (LDCs), two specific concerns rapidly come into play when examining such projects. First, attracting foreign investors is a tricky task when the proposed infrastructure is aimed at supplying natural gas into a region where commercial energy consumption has hitherto remained limited. To overcome this, development agencies recurrently advocate the need to "build an anchor load" (e.g., ESMAP, 2007; World Bank, 2010; ICF, 2012) by strategically locating a small group of creditworthy industrial gas users at the outlet of the proposed pipeline. It is expected that this load will be sufficient to trigger the investment decision and that, in the aftermath of the opening of the pipeline, a supplementary demand emanating from local users will naturally emerge. However, this approach implicitly presumes that the installed pipeline capacity will be sufficient to serve the enlarged demand which, in turn, supposes that investors agree to "build ahead of demand" by installing some extra capacity that may remain superfluous for a number of years after the opening of the infrastructure. Second, the energy delivery must remain affordable so as to maximize the development benefits. Given the natural monopolistic essence of a pipeline infrastructure, there is a need to mitigate the private pipeline operator's ability to exert market power by implementing an adapted form of regulation. Against this background, a question of some policy relevance

for developing nations is therefore: can the two public policy objectives of attracting an adequate amount of investment and protecting society from monopolistic prices be jointly attained?

In this paper, we investigate this question for a form of regulation commonly implemented in developing nations: the rate-of-return (RoR) regulation. That method is one of the simplest form of regulation: it allows the regulated company to cover its operating and capital costs and to earn a return on capital. Its roots are generally identified with the regulation applied to investor-owned utilities in the US. Its shortcomings are extensively discussed in the literature (see, for example, Laffont and Tirole, 1993). An important effect that was first highlighted in Averch and Johnson (1962) concerns the tendency of regulated firms to engage in excessive amounts of durable capital accumulation. One can thus wonder whether that overcapitalization – which is recurrently presented as a limitation in the literature – could represent a blessing in the specific case of a gas pipeline project located in a developing country.

The purpose of this paper is to methodically examine the economics of the "anchor load" strategy in a developing country where RoR regulation has been implemented. More precisely, we investigate whether it is possible to leverage on the overcapitalization behavior of the regulated firm to obtain *ex ante* (i.e., at the planning stage) the adapted degree of capital installation that will be needed *ex post* (i.e., after the opening of the infrastructure) to serve the envisioned larger demand in a cost-efficient manner.

Our point of departure is in the literature on RoR regulation (Klevorick, 1971; Callen et al., 1976). Using an engineering-based representation of the gas pipeline technology, we first show that the allowable rate of return is a control variable for the degree of *ex-ante* overcapitalization decided by the regulated firm. Building upon this remark, we then characterize the behavior of that operator in the case of a larger *ex-post* demand. In particular, we derive the conditions for the regulated firm to serve that enlarged demand in a cost-efficient manner. This leads us to analytically prove that the allowable rate of return can be tuned to influence the investors' planning decisions so as to obtain the provision of an optimal infrastructure *ex post*. Lastly, we examine the impacts of that strategy on the net social welfare.

Though our approach is motivated by the case of a real pipeline project in Mozambique, we believe that our analysis is not specific to that case and could generally inform the infrastructure development strategies pursued in other developing nations.

The paper is organized as follows: Section 2 reviews the main features of a pipeline development project examined in Mozambique with the aim to both clarify the motivation for our analysis and justify our assumptions. Section 3 presents our modeling framework. In section 4, we examine whether RoR regulation can jointly satisfy the two public policy objectives of building ahead of demand and limiting the market power exerted by the monopoly. Finally, the last section offers a summary and some concluding remarks.

2. Background: A Mozambican pipeline project

In this section, we use the case of a real project in Mozambique to review the main specificities governing the provision of natural gas infrastructures in developing countries. After a brief presentation of that context, we discuss the limitations of the literature supporting the policy recommendation to build the infrastructure ahead of proven demand. Finally, we present the regulatory framework governing the provision of these infrastructures.

2.1 The Mozambican natural gas scene

Emergence of a gas-fired economy

In Mozambique, a series of recent natural gas discoveries in the Rovuma basin radically changed the country's resource endowment[1] and their monetization through liquefied natural gas (LNG) exports is expected to make the country a large energy exporter (Mahumane and Mulder, 2016) and to move the economy away from a low-development trap (Melina and Xiong, 2013; IMF, 2016). Yet, LNG exports alone cannot solve all the country's development problems as they promise few permanent jobs and generate little forward and backward linkages à la Hirschman (1958) with the rest of the economy. To improve the well-being of the population, the government has also revealed its ambitions to leverage on its subsoil wealth by allocating a share of its royalty gas to the national market to promote both industrialization and the domestic use of natural gas (ICF, 2012; Ministério da Planificação E Desenvolvimento, 2014).

[1] *According to the 2014 Oil & Gas Journal annual survey, Mozambique's proved natural gas reserves amount to 100 trillion cubic feet (Tcf) – compared to 4.5 Tcf the previous year – and are now the third largest in Africa (Xu and Bell, 2013).*

The first aspect of the Mozambican plan consists of attracting FDI into the so-called "mega-projects": a small number of large-scale, export-oriented, gas-based industries that process natural gas into fertilizers, petrochemicals or direct reduced iron. These projects are expected to generate forward and backward linkages[2] and to moderate the variance of the country's export revenues by diversifying commodity price risk away from gas (as in Massol and Banal-Estañol, 2014).

The second aspect of that plan emphasizes: (i) the supply of natural gas to domestic-oriented uses (e.g., local businesses, cement manufacturers, households) to promote job-creation; (ii) the substitution of expensive and imported oil products (ICF, 2012); and (iii) the development of gas-fired thermal generation in the Northern provinces to support the government's electrification plans (ICF, 2012).

Overcoming the country's pipeline deficit

Geography rapidly comes into play when assessing the feasibility of that plan. The Rovuma gas fields are located offshore in rural and scarcely populated districts. This remote location imposes the construction of a pipeline system to connect the fields to the country's main population centers (see Figure 1).

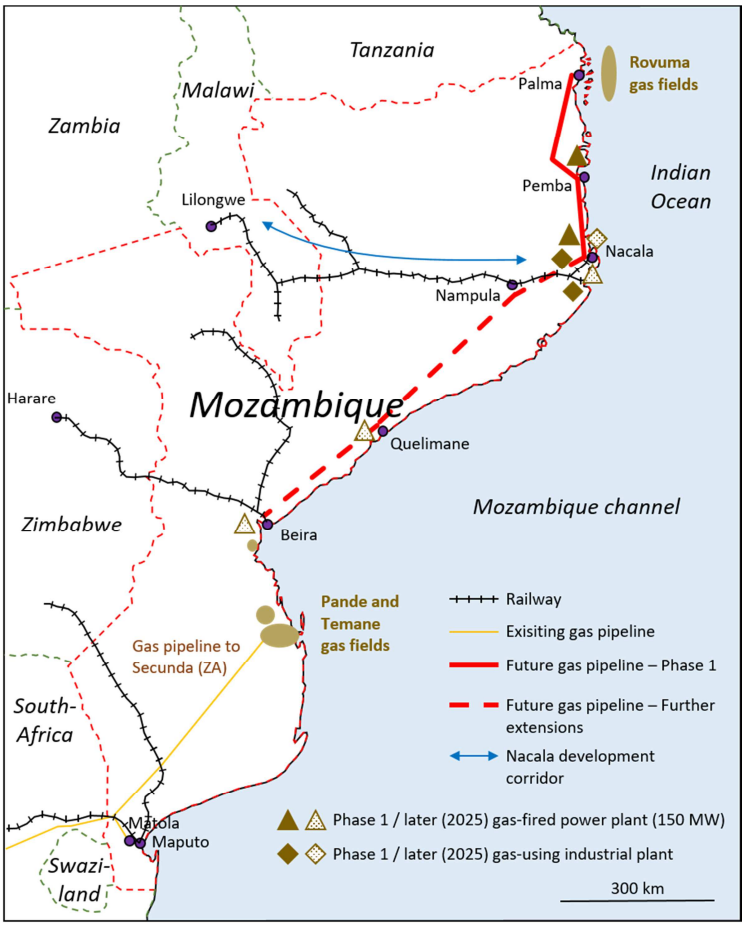
However, implementing a pipeline infrastructure is a classic instance of a "chicken and egg" problem. It is not worth building an expensive pipeline system without a critical mass of consumers capable of supporting the construction of the infrastructure and, without the pipeline, the potential demand from users is unlikely to materialize. In Mozambique, the problem is trickier because the domestic-oriented, gas-consuming sectors have to be developed from a very low existing base (IEA, 2014) which makes them unlikely to attain that critical size in the foreseeable future.[3] To overcome this problem, development planners envision leveraging on the FDI-financed mega-projects to facilitate the deployment of this pipeline infrastructure (ICF, 2012; IEA, 2014). Rather than allowing their constructions in Palma (Figure 1), that approach consists of strategically locating them in the deep-port city of Nacala that provides a larger development potential. The city is the marine terminal of an agricultural production area which is home to approximately 10 million people: the Nacala Development Corridor that reaches westward from Nacala to

[2] *For example, a fertilizer industry is expected to foster the modernization of the country's agricultural sector which is currently dominated by subsistence farming with a scant use of fertilizers (Franza, 2013).*

[3] *For example, Jones and Tvedten (2019) underline that close to half of all Mozambicans live in households who cannot afford a minimal basket of goods that meets their basic needs.*

landlocked Malawi (Figure 1). Locating the mega-projects there would provide the "anchor" load needed to justify the construction of a pipeline system along the northern coastline from Palma to Nacala (ICF, 2012).[4,5] In recognition of this, the government is actively trying to promote FDI in Nacala and has created a special economic zone aimed at providing fiscal incentives and guarantees to foreign investors.

Figure 1. Map of Mozambique's northern pipeline deployment



2.2 Building a pipeline ahead of demand?

The literature on natural gas pipeline provides two interesting insights that have important implications for the planning of such investments. First, the technology of a natural gas pipeline exhibits pronounced

[4] *Indeed, before installing immobile gas-processing assets in Nacala, the promoters of mega projects typically sign binding long-term supply contracts aimed at organizing the delivery of a predefined volume of natural gas to their plant.*

[5] *In addition to fostering domestic-oriented uses in the Nacala region, this pipeline could also unlock a series of future pipeline deployment phases. The Palma-to-Nacala route could be integrated within a longer pipeline system reaching first the cities of Quelimane and Beira and ultimately the capital Maputo and the South African market (ICF, 2012). It could also be integrated within a broader transnational pipeline infrastructure such as the one examined in Demierre et al. (2015).*

increasing return to scale in the long run (Kahn, 1988, vol. II, p. 153). Second, investment in a natural gas pipeline conveys some irreversibility. *Ex ante*, during the planning phase, investors can use any combination of pipe diameter and compressor horsepower, as long as the corresponding engineering constraints are observed. However, once installed, the diameter of a pipeline – and thus the capital stock that has been immobilized – can no longer be modified without incurring prohibitive costs.[6] Because of this irreversibility, any *ex post* rise in output must be accommodated by adjustments in the compression horsepower. Because of the joint presence of irreversibility and pronounced economies of scale, the literature shows that, in the case of an output level that can rise in the future, it is rational to "build ahead of demand" and install an optimum degree of overcapacity (in the form of a larger pipeline diameter than the one installed in case of zero future increment in output) to minimize the present value of the infrastructure's total cost (Chenery, 1952; Manne, 1961; Massol, 2011).

While building ahead of demand has been used for decades in the industrialized world, it is important to stress that the forces supporting its application in developing countries are much weaker. A project developer typically faces the dilemma of building ahead of demand – and thus operating with a low degree of capacity utilization during the early years – or installing a less-than-optimal-size pipeline system. For a private sector-driven project,[7] this has important implications because the capital needs of the infrastructure have to be financed by the project's future cash flow stream. In a developing country, the margin of errors affecting the estimate of future buying decisions by low-income customers can be substantial, and the limited financial viability of the project firm may not allow it to err significantly. If the operator ends up having installed too much excess capacity, the project's output will be concentrated on a portion of the consumer base that is too small to allow a recovery of the infrastructure cost (Hirschman, 1970). Another explanation for the difficult implementation of "building ahead of demand" is given by arguments based on real options theory. Because of these uncertainties, laying a pipeline with an oversized diameter car be described as undertaking an

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[6] *The pipe diameter is reputed to give an index of the size of the infrastructure (Chenery, 1952).*

[7] *For concision, we do not discuss the case of "building ahead of demand" in public sector driven projects. During the 1960's, a number of developing nations initiated large, government-controlled, infrastructure projects. At that time, governments attempted to replicate the western countries' experiences with "building ahead of demand" by installing oversized equipment but subsequent analyses revealed that these projects were cursed by, among others, bureaucracy and politics (Hirschman, 1970).*

irreversible decision conveying important opportunity cost (i.e., the "value of waiting") and that it can be preferable to defer that investment decision (Pindyck, 1991).

2.3 Regulatory framework

In light of both the scarcity of domestic capital and the poor performance of state-owned enterprises in developing countries (Parker and Kirkpatrick, 2005), the participation of an FDI-financed private sector is needed to provide the energy infrastructure. However, the natural gas pipeline sector has "natural monopoly" features because its long-run cost function is strictly subadditive (Perrotton and Massol, 2018). As that characteristic can lead to a variety of economic performance problems (i.e., excessive prices, production inefficiencies), some form of economic regulation is necessary.

In a comprehensive review, Joskow (1999) examines how effective regulatory institutions can be established in a developing nation. He stresses that the regulatory framework should be adapted to take into account the presence of weak institutions and the lack of regulatory and antitrust expertize. He also highlights that, in the case of a nascent infrastructure sector, it is preferable to implement simple procedures. Mozambique followed these recommendations and opted for a simple and proven form of regulation for its natural gas pipeline sector: RoR regulation.[8] That form of regulation sees costs as exogenous and observable and forms prices on the basis of observed costs and a predetermined appropriate rate of return on the investments. By construction, it is well suited to attract foreign investment as it provides investors with a reasonable opportunity to recover investment and operating costs as well as a return on capital.

The shortcomings of RoR regulation are extensively discussed in the literature, though, and were first presented in Averch and Johnson (1962). The main reservations against this approach are that it does not provide incentives for cost savings and efficiency improvements, and that it rewards an excessive investment in fixed assets. This so-called Averch-Johnson effect calls for a condemnation of the tendency of regulated firms to engage in excessive amounts of durable capital accumulation to expand the volume of their profits.[9]

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[8] *This form of regulation has been extensively used to regulate privately owned pipeline infrastructures in the US and in a neighboring country: South Africa.*

[9] *A possible remedy for this effect can consist of the adoption of a regulatory control over the input choice (Laffont and Tirole 1993).*

For the Mozambican project, this effect could play a positive role, though. The discussion above suggests that while governmental planners could wish to encourage some degree of "building ahead of proven demand" to cost-efficiently supply the future flows of gas consumed in Mozambique, they may have a hard time convincing the pipeline's foreign investors to immobilize the extra amount of capital needed to serve an embryonic domestic market whose future take-off is far from being granted. Interestingly, the Averch-Johnson effect suggests that a myopic (or conservative) profit-maximizing operator subject to RoR regulation (i.e., a firm that totally ignores the evolution of the domestic-oriented uses and bases its decisions solely on the natural gas demand from the FDI-financed mega-projects) can rationally decide some degree of overcapitalization. In the next section, we will thus show that under certain conditions planners could leverage on the behavior of the regulated firm to induce the installation of an appropriate degree of overcapacity.[10]

3. Model

In this section, we prove that it is possible to adapt the parameters of rate-of-return regulation to a potential demand growth so as to encourage building ahead of demand and foster efficient pipeline operation. We first introduce the notation and clarify our assumptions. We then successively examine: (i) the *ex-ante* situation to identify the investment planning decisions taken by a regulated operator, and (ii) that firm's reaction to an *ex-post* demand growth. Lastly, having shown that it is possible to induce *ex ante* an efficient degree of building ahead of demand by adjusting the allowed rate of return, we examine the implications.

For the sake of conciseness, all the mathematical proofs are presented in Appendix A.

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[10] *One can note that in other infrastructure sectors, the overcapitalization can result in the accumulation of a large asset base that can include both productive capital equipment and non-productive one (e.g., an accumulation of non-core assets). In our specific case of a simple point-to-point pipeline infrastructure, this concern can be mitigated by remarking that there exists a one-to-one relation between the amount of capital invested in the pipeline and an easily observable technical parameter: the pipeline diameter (Perrotton and Massol, 2018). That relation makes it straightforward for a regulatory agency to verify whether the overcapitalization has resulted in an increased pipeline capacity or not. Another approach routinely used by regulators is to mobilize detailed engineering models such as the one proposed in Yépez (2008). These models are routinely used by development planning agencies to verify whether the engineering parameters used by the promoters of a pipeline system are consistent or not with the desired output of the infrastructure.*

3.1 Assumptions and notations

Institutional organization

We assume, as in Mozambique, that the regulatory and institutional framework governing the natural gas pipeline sector has the three following characteristics. First, the natural gas pipeline (i.e., the midstream sector) is treated as a vertically separated entity from the rest of the supply chain. The pipeline firm does not own the natural gas it transports and simply provides a point-to-point transportation service. Second, the pipeline firm's profitability is restricted by the application of the RoR regulation. Third, the regulation institutes a transparent open-access regime that obliges the pipeline firm to charge a non-discriminatory price per unit transported.

Technology

A simple point-to-point pipeline system consists of a compressor station injecting a pressurized flow of natural gas Q into a pipeline to transport it across a given distance. Using an engineering approach, Perrotton and Massol (2018)[11] recently proved that the technology of that infrastructure can be approximated by a single production equation of the Cobb-Douglas type:

$$Q^{\beta} = K^{\alpha} E^{1-\alpha}, \tag{1}$$

where E is the amount of energy consumed by the infrastructure to power the compressor, K is the capital stock employed, $\alpha = 8/11$ is the capital exponent parameter, and $\beta = 9/11$ is the inverse of the degree to which output is homogeneous in capital and energy. As $\beta < 1$, the technology exhibits increasing returns to scale.

From that production function, one can define $E(Q,K) = \sqrt[1-\alpha]{K^{-\alpha} Q^{\beta}}$ the variable input requirements function that gives the amount of energy needed to transport the output Q on a pipeline infrastructure that has a given fixed amount of capital input K. We let $E_Q(Q,K)$ (respectively, $E_K(Q,K)$) denote the

[11] *For clarity, a shortened presentation of the derivation of that engineering-based production function is also presented in a Supplementary Document.*

derivative of the input requirement function with respect to the output (respectively, the capital) variable. With our technology parameters, $E_Q(Q,K)>0$ and $E_K(Q,K)<0$.

Costs

We let e denote the market price of the energy input and r the market price of capital faced by the firm. Following Perrotton and Massol (2018), the long-run, cost-minimizing amount of capital stock needed to transport the flow Q is:

$$K(Q)=\left(\frac{e\alpha}{r(1-\alpha)}\right)^{1-\alpha}Q^{\beta}, \tag{2}$$

and the long-run, total cost function is $C(Q)=rK(Q)+eE(Q,K(Q))$ which after simplification gives:

$$C(Q)=\frac{r^{\alpha}e^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}Q^{\beta}. \tag{3}$$

This presentation naturally leads to the following definition.

***Definition*:** *The capital-output combination* (K,Q) *is cost-efficient if the capital stock* K *equals the long-run, cost-minimizing amount of capital stock needed to transport the flow* Q*, that is:* $K=K(Q)$.

Ex ante demand

At the planning stage (i.e., before the construction of the infrastructure), we assume the *ex-ante* demand schedule for pipeline transportation services. This demand is the "anchor load" that emanates from the large users that are planning to install their gas-based processing activities at the outlet of the pipeline. This *ex-ante* inverse demand function is:

$$P(Q)=A\,Q^{-\varepsilon}, \tag{4}$$

where A is a constant and the constant $1/\varepsilon$ is positive and denotes the absolute value of the price elasticity of demand. We assume that $\varepsilon<1$ so that the total revenue obtained by an unregulated pipeline operator

producing zero output is zero and that $\varepsilon > 1-\beta$ so that the demand schedule always intersects the marginal cost schedule from above.[12]

For notational convenience, we follow Callen et al. (1976) and introduce three parameters: (i) $\gamma \equiv \beta+\varepsilon-1$, (ii) $\delta \equiv e\beta/\left[A(1-\varepsilon)(1-\alpha)\right]$, and (iii) $\eta \equiv \beta-(1-\varepsilon)(1-\alpha)$.

<u>The cases of a monopoly and of a social planner</u>

To gain insight into the performance of the regulated firm, in Table 1 we summarize the market outcomes obtained under two polar cases: (i) the profit-maximizing, unregulated monopoly (column 1); and (ii) a welfare-maximizing social planner that maximizes the sum of the producers' and consumers' surpluses (i.e., the net social welfare) while ensuring zero economic profit for the pipeline activity[13] (column 2). These outcomes are subscripted with M and a respectively and are presented in Table 1. In both cases, production is cost-efficient and uses the cost-minimizing amount of capital stock, that is, $K_M = K(Q_M)$ and $K_a = K(Q_a)$. Note also that, for the social planner, substitution of the optimal decisions Q_a and K_a in the zero profit condition $P(Q)Q - rK - eE(Q,K) = 0$ gives $P(Q_a)Q_a - C(Q_a) = 0$ which means that the output Q_a is set at a level such that the price equals the long-run average cost.

Table 1. The cases of a profit-maximizing, unregulated monopoly and a welfare-maximizing social planner providing zero profit to the firm

[Please insert Table 1 here]

Following Callen et al. (1976), we define s_M the unregulated monopolist's rate of return on invested capital obtained by evaluating the accounting profit $P(Q_M)Q_M - eE(Q_M, K_M)$ and dividing it by K_M the profit-maximizing capital stock. After simplification, the unregulated monopolist's rate of return is: $s_M \equiv \eta r/\left[\alpha(1-\varepsilon)\right]$.

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[12] *For $\beta = 9/11$, these two restrictions indicate that the price elasticity of the demand is in the range (-5.5, -1.0).*

[13] *Recall that the first-best solution obtained under an unconstrained welfare maximizer would not allow the pipeline activity to break-even. This organization thus resembles the second-best solution examined in Boiteux (1956).*

3.2 The ex-ante behavior of the regulated firm

We now assume that the infrastructure is provided by a monopolistic private operator that is subject to RoR regulation and examine the *ex-ante* situation (i.e., at the planning stage, before the construction of the infrastructure). As that situation is similar to the one examined in Klevorick (1971) and Callen et al. (1976), we briefly review the results gained in these two early contributions.

The regulated monopoly is allowed to earn an exogenously-determined rate of return s on the invested capital K. The RoR constraint stipulates that the monopoly's accounting profit, defined as the difference between the total revenue $P(Q)Q$ and the cost of the variable input $eE(Q,K)$, cannot exceed the allowed return on invested capital sK. In the sequel, we assume that s is not greater than the rate of return s_M obtained by an unregulated monopolist (i.e., $s \leq s_M$ holds) so that the RoR constraint is binding and can be written as the equality constraint:

$$P(Q)Q - e\,E(Q,K) = s\,K, \tag{5}$$

The regulated monopoly's problem amounts to determining the combination of capital K and output Q that: (i) maximizes its profits (i.e., the difference between the total revenue $P(Q)Q$ and the sum of the total cost of capital rK plus $eE(Q,K)$ the total cost of energy), and (ii) verifies the RoR constraint. That optimization program is presented in Table 2 – panel 1.

Table 2. The *ex-ante* behavior of the regulated firm

[Please insert Table 2 here]

Klevorick (1971) provides a detailed examination of that optimization problem. He proves that the first-order optimality conditions of the firm's problem are such that the optimal pair (K^*, Q^*) must jointly verify the RoR constraint (5) and the following condition:

$$(s-r)\left[P'(Q)Q + P(Q) - e\,E_Q(Q,K)\right] = 0, \tag{6}$$

where $E_Q(Q,K)$ is the derivative of the input requirement function with respect to the output variable.

If $s < r$, the allowed RoR is lower than the market price of capital and the firm's optimal decision is to withdraw from the market. To eliminate this corner solution, we hereafter concentrate on the more interesting case $r \leq s \leq s_M$ whereby the allowed RoR is not lower than the cost of capital.

If $s = r$, the constraint (6) is *de facto* verified. The behavior of the regulated monopoly is thus indeterminate because any capital and output combination that verifies the rate-of-return constraint (5) – i.e., which yields zero profit – can be considered by the firm.[14] To avoid that indeterminacy, we thereafter prohibit setting s equal to the market price of capital.

If $r < s \leq s_M$, the condition (6) is equivalent to $P'(Q)Q + P(Q) - eE_Q(Q,K) = 0$ which is the analogue for a regulated monopoly of the standard condition for profit maximization: the marginal revenue $P'(Q)Q + P(Q)$ has to be equal to the regulated marginal cost $eE_Q(Q,K)$, which is the marginal cost to produce an additional unit of output when K is set at the level required to satisfy the RoR constraint (5). Callen et al. (1976) analytically determine the unique capital and output combination (K^*, Q^*) that verifies both the condition $P'(Q)Q + P(Q) - eE_Q(Q,K) = 0$ and the RoR constraint. That solution is presented in Table 2 – panel 2.

To examine the implications, Callen et al. (1976) detail: (i) the total cost C^* incurred by the regulated firm; (ii) the net social welfare W^*; (iii) the output ratio Q^*/Q_M that measures the relative increase in output effected by imposing a regulated rate of return on an unregulated monopoly; (iv) the capital ratio $K^*/K(Q^*)$ that provides a relative measure of the capital stock employed by the regulated firm K^* with respect to $K(Q^*)$ obtained using (2) the capital stock that would have been installed by a cost-minimizing firm producing the same output; and (v) the cost ratio $C^*/C(Q^*)$ that compares the total cost incurred by the regulated firm C^* and the total cost $C(Q^*)$ that would have been incurred to serve the output Q^* if

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[14] *If $s = r$, each of the three following capital-output pairs yields zero economic profit and thus verifies the RoR constraint: $(0,0)$, the pair (K_a, Q_a) in Table 1 that maximizes the net social welfare while giving zero profits to the firm, and the pair (K^*, Q^*) in Table 2 – panel 2.*

production had used a cost-minimizing combination of inputs. For the sake of brevity, these values and ratios are presented in Table 2 – panel 3 and we simply mention below the value of the capital ratio $K^*/K(Q^*)$:

$$\frac{K^*}{K(Q^*)} = \left(\frac{r}{s}\frac{\eta}{(1-\varepsilon)\alpha}\right)^{1-\alpha}. \tag{7}$$

The following lemma indicates that this ratio is inversely related to the allowed rate of return.

Lemma 1**:** *The capital ratio* $K^*/K(Q^*)$ *is a smooth and monotonically decreasing function of* s/r *, the ratio of the allowable rate of return to the cost of capital. Hence, there is a one-to-one mapping between* $s \in (r, s_M]$*, the range of admissible values for the allowed rate of return, and* $\left[1, \lim_{s \to r} K^*/K(Q^*)\right) = \left[1, \left(\eta/[(1-\varepsilon)\alpha]\right)^{1-\alpha}\right)$*, the range of feasible values for the overcapitalization ratio* $K^*/K(Q^*)$.

For concision, we omit the straightforward proof of this lemma but rather emphasize its economic implications. If the allowed rate of return s (with $s > r$) is lower than the rate of return obtained by an unregulated monopoly (i.e., $s < s_M$), the profit-maximizing regulated firm selects a capital-output pair (K^*, Q^*) which is not cost-efficient because the value of the capital ratio is greater than one. This is the overcapitalization distortion pointed in Averch and Johnson (1962). This lemma also indicates that the allowed rate of return s is a control variable for the magnitude of that overcapitalization.

3.3 The ex-post behavior of the regulated firm

Having observed the allowed rate of return s with $r < s \leq s_M$, the regulated firm has installed the capital stock K^* to transport the pipeline throughput Q^*. We now provide an original contribution to examine the *ex-post* situation after the opening of the infrastructure.

Following the discussion in section 2, we assume that an expanded demand for gas transportation services is observed *ex post*.[15] So, the pipeline operator now faces the *ex-post* inverse demand function:

$$P_{\lambda}(Q) = (1+\lambda) A Q^{-\varepsilon}, \tag{8}$$

where λ is a positive parameter reflecting that demand expansion.

Two important features have to be noted. First, the regulatory authority cannot renege on the allowed rate of return s with $r < s \leq s_M$ that was set prior to the construction of the pipeline. Second, investment in a pipeline has an irreversible nature: once installed, the diameter of a pipeline can no longer be modified without incurring prohibitive costs. So, the capital stock employed by the firm is fixed and maintained at the *ex-ante* value K^*. Hence, any *ex-post* change in output is accommodated solely by adjustments in the variable input: energy.

The firm must verify the *ex-post* rate-of-return constraint:

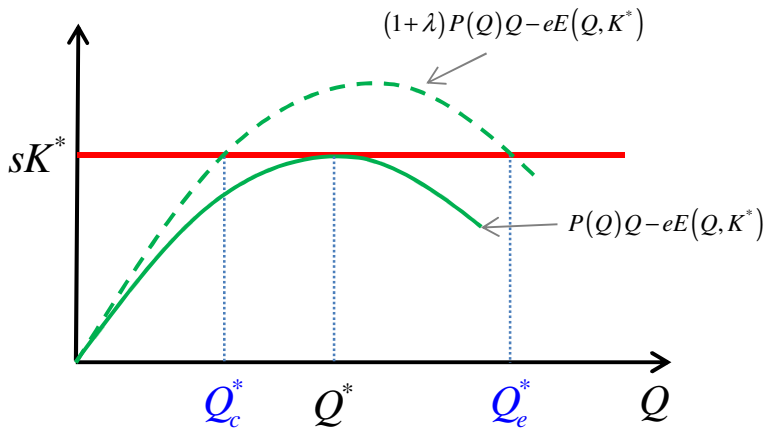
$$(1+\lambda) P(Q) Q - eE(Q, K^*) = sK^*. \tag{9}$$

As $\lambda > 0$, the output level Q^* chosen *ex ante* does not verify the *ex-post* rate-of-return constraint. (Inserting Q^* in (9) and using the *ex-ante* rate-of-return constraint (5) yields the equation $\lambda P(Q^*) Q^* = 0$ which cannot hold because Q^* is positive). So, the regulated firm has to adjust its output level *ex post*. To overcome that problem, the following proposition indicates that the firm can either consider a contraction of its output down to the level Q_c^* or an expansion up to Q_e^*.

Proposition 1: *If $\lambda > 0$, there exists exactly two output levels: Q_c^* and Q_e^*, such that the ex-post rate-of-return constraint* (9) *is verified. These two output levels verify $Q_c^* < Q^* < Q_e^*$.*

[15] *For example, small users (e.g., small and medium enterprises) that were overlooked at the planning stage may substitute expensive heating oil for natural gas once it becomes available. The possibility of such an ex-post expansion of the pipeline output after the opening of the infrastructure is frequently discussed in policy analyses (e.g., Sovacool, 2009).*

Figure 2. The *ex-post* behavior of the regulated firm with $r < s < s_M$



An illustration is presented in Figure 2. It shows the value of the regulatory constraint sK^*, which is constant as the capital stock K^* is fixed, and two curves in green. The solid curve represents how the firm's accounting profit *ex ante* – i.e., $P(Q)Q - eE(Q,K^*)$, the difference between the total revenue minus the total variable cost – varies with the firm's output level. The developments above have shown that Q^* is the unique output such that the *ex-ante* accounting profit equals the allowed value sK^*. The dotted curve illustrates the *ex-post* case. It represents the *ex-post* accounting profit $(1+\lambda)P(Q)Q - eE(Q,K^*)$. In that case, two output levels verify the *ex-post* RoR constraint (9).

As the *ex-post* behavior of the regulated firm is indeterminate, it is instructive to confront the two candidate solutions with the context presented in section 2. In a developing country, the relationships between the gas producers, the pipeline operator and the large industrial users (i.e., the "mega projects") are typically governed by specific long-term bilateral contracts. These contracts are signed *ex ante* (i.e., before the construction of the infrastructure and the industrial sites)[16] and traditionally include minimum "take-or-pay" obligations that: (i) compel the industrial user to purchase at least the contracted quantity, (ii) commit

[16] *For a multinational gas-processing firm, the decision to locate a mega project in a given country has elements of a relationship-specific asset. Once investment in that gas-processing plant is sunk, there exists appropriable specialized quasi rents à la Klein et al. (1978). If transactions between that firm and the gas suppliers are governed by "simple" short-term contracts, asset-specific investments and uncertainty imply high transaction costs that can jeopardize the feasibility of the transaction and thus the installation of the plant. In such situations, as full vertical integration is not credible, transaction costs can be reduced by signing long-term contracts ex ante (Williamson, 1983). These contracts include requirement clauses, liquidation damages, arbitration, pricing, and other provisions.*

the producer to supplying at least that quantity, and (iii) compel the pipeline firm to transport that quantity. Because of these contractual arrangements, a contraction of the transported flow of natural gas below the output level Q^* is unlikely. Against this backdrop, an expansion of the pipeline output up to the level Q_e^* represents the preferred option.[17]

In general, it is not possible to determine a closed-form expression for Q_e^* as a function of the technology and demand parameters. Nevertheless, the following corollary clarifies how the output level Q_e^* varies with the *ex-post* demand expansion coefficient λ.

Corollary 1**:** *The output Q_e^* (respectively, Q_c^*) is monotonically increasing (respectively, decreasing) with the demand parameter λ.*

It should be noted that the results above (in Proposition 1 and Corollary 1) jointly provide a characterization of the output level Q_e^* as the unique output level that both verifies the *ex-post* rate-of-return constraint (9) and is monotonically increasing with the demand parameter λ.

3.4 Installing ex ante an appropriate degree of overcapitalization

So far, our analysis has highlighted two results. First, section 3.2 confirms the tendency of the regulated firm to engage in excessive amounts of capital accumulation at the planning stage. Second, section 3.3 shows that, once the infrastructure is in place, the materialization of a larger demand imposes the regulated firm to expand its output beyond the planned level. So, we now have to examine whether that *ex-post* output could be large enough to "absorb" the larger-than-needed amount of capital stock immobilized *ex ante*. In other words, we have to explore whether, in the case of an initial demand underestimation by the regulated firm, the *ex-ante* overcapitalization could provide an opportunity to optimally install the amount of capital stock needed to transport the *ex-post* output in a cost-efficient manner.

---

[17] *Of course, in the case of a very large λ, one could question the feasibility of the output expansion without taking into consideration the technical constraints that govern the mechanical stability of a pressurized pipeline. Yet, a series of discussions with technical experts have convinced us that the influence of these technical considerations can be omitted for the range of λ considered in the present analysis.*

To examine this, we first focus on the *ex-post* situation and derive a closed-form expression for the capital-output combination that is cost-efficient and verifies the *ex-post* rate-of-return constraint. Then, we clarify the conditions under which the regulated firm would install *ex ante* that desired level of capital stock.

<u>The cost-efficient, capital-output combination that verifies the *ex-post* rate-of-return constraint</u>

We consider a cost-efficient, capital-output combination (K_{ce}, Q_{ce}) that also verifies the *ex-post* rate-of-return constraint:

$$(1+\lambda)P(Q_{ce})Q_{ce} - eE(Q_{ce}, K_{ce}) = sK_{ce}, \tag{10}$$

where s is the given allowed rate of return with $r < s \leq s_M$.

As (K_{ce}, Q_{ce}) is cost-efficient, one can use (2) and replace the capital stock K_{ce} by $K(Q_{ce})$. Subtracting the total cost of capital $rK(Q_{ce})$ on both sides of that equation and remarking that the total cost $rK(Q_{ce}) + E(Q_{ce}, K(Q_{ce}))$ equals the long-run total cost of transporting the flow Q_{ce}, that is, $C(Q_{ce}) = rK(Q_{ce}) + E(Q_{ce}, K(Q_{ce}))$, we obtain:

$$(1+\lambda)P(Q_{ce})Q_{ce} - C(Q_{ce}) = (s-r)K(Q_{ce}). \tag{11}$$

Substituting equations (2), (3), and (4) into (11) and solving that single-variable equation yields the output level Q_{ce} such that the capital-output combination $(K(Q_{ce}), Q_{ce})$ is cost-efficient and verifies the *ex post* rate-of-return constraint:

$$Q_{ce} = \left[\frac{(1+\lambda)A}{\left(\frac{s}{r}-1\right)\alpha+1} \left(\frac{\alpha}{r}\right)^{\alpha} \left(\frac{1-\alpha}{e}\right)^{1-\alpha} \right]^{1/\gamma}. \tag{12}$$

Using equation (2), the amount of capital stock that minimizes the long-run cost of transporting that output is $K_{ce} = K(Q_{ce})$, that is:

$$K_{ce} = \left(\frac{e\alpha}{r(1-\alpha)} \right)^{1-\alpha} \left[\frac{(1+\lambda)A}{\left(\frac{s}{r} - 1 \right)\alpha + 1} \left(\frac{\alpha}{r} \right)^{\alpha} \left(\frac{1-\alpha}{e} \right)^{1-\alpha} \right]^{\beta/\gamma} . \tag{13}$$

Two remarks can be formulated on the cost-efficient capital-output combination (K_{ce}, Q_{ce}). First, by construction, that combination verifies the *ex-post* RoR constraint (10). So, when the regulated firm *ex ante* installs an amount of capital stock K^* equal to K_{ce}, the discussion in section 3.3. shows that the output level Q_{ce} represents one of the two candidate solutions that can be considered *ex-post* by the regulated firm. In the sequel, we are going to clarify whether Q_{ce} corresponds to either the expansion or the contraction case. For the moment, we simply keep in mind that Q_{ce} has an interesting feature: it is a candidate solution that has a closed-form expression.

Second, it should be noted that the cost-efficient capital stock K_{ce} in (13) is parameterized by the allowed rate of return s. Recall that, following Lemma 1, we have already noted that K^* the optimal amount of capital stock decided by the regulated firm *ex ante* is also parameterized by s. One may thus wish to explore whether that rate s could be set at a level such that the regulated firm rationally decides to install *ex ante* the amount of capital stock K^* that equals K_{ce}.

<u>Obtaining *ex ante* the installation of the cost-efficient amount of capital stock</u>

We now explore the condition for the regulated firm to rationally decide to immobilize the capital stock K^* that equals the cost-efficient level $K_{ce} = K(Q_{ce})$ presented in (13).

Using equation (2), we can write the following equation $K(Q_{ce}) = (Q_{ce}/Q^*)^{\beta} K(Q^*)$. Introducing the output level Q_M chosen by an unregulated monopoly facing the inverse demand function (4), that equation suggests that the condition $K^* = K_{ce}$ is logically equivalent to:

$$\frac{K^*}{K(Q^*)} \times \left(\frac{Q^*}{Q_M} \times \frac{Q_M}{Q_{ce}} \right)^{\beta} = 1 . \tag{14}$$

In this condition, one can readily identify the two ratios introduced in Callen et al. (1976) and reviewed in section 3.2: the output ratio Q^*/Q_M in Table 2 (equation VI) and the capital ratio $K^*/K(Q^*)$ in Table 2 (equation VII). The ratio Q_M/Q_{ce} is easy to evaluate using the value of the output level of the unregulated monopolist Q_M indicated in Table 1 (equation IV) and that of Q_{ce} detailed in (12). Substituting these results into equation (14) and simplifying, the condition for a firm facing a rate of return s and a given demand expansion factor λ to install *ex-ante* an amount of capital equal to the *ex-post* cost-efficient amount of capital becomes:

$$\frac{(1-\varepsilon)}{\beta}\left[\left(\frac{s}{r}-1\right)\alpha+1\right]\left(\frac{r}{s}\frac{\eta}{(1-\varepsilon)\alpha}\right)^{\eta/\beta}-1=\lambda\,. \tag{15}$$

The following proposition clarifies the conditions for that equation to hold.

Proposition 2: *For any demand expansion coefficient λ such that $0<\lambda<\overline{\lambda}$ where the upper bound is $\overline{\lambda}\equiv\left[\frac{\eta}{(1-\varepsilon)\alpha}\right]^{\frac{\eta}{\beta}}\left[\frac{1-\varepsilon}{\beta}\right]-1$, there exists a unique allowed rate of return s_λ in the open interval (r,s^M) such that the regulated firm rationally decides to install ex ante the amount of capital stock K^* compatible with the cost-efficient capital-output combination $(K(Q_{ce}),Q_{ce})$ that verifies the ex-post regulatory constraint (i.e., the amount of capital stock K^* that verifies the condition (15)). Moreover, the allowed rate of return s_λ is monotonically decreasing with the demand parameter λ.*

So, if the *ex-post* demand expansion coefficient λ is lower than the upper bound $\overline{\lambda}$, and if the regulator sets the allowed rate of return at the level s_λ, the regulated firm's best response to that rate is to *ex ante* install the capital stock $K^*=K(Q_{ce})$. Proposition 1 indicates that, *ex post,* the regulated firm must adjust its output to verify the *ex-post* rate-of-return constraint (9) by either raising it to Q_e^* or lowering it to Q_c^*. By construction, the cost-efficient output level Q_{ce} verifies that *ex-post* constraint and is thus equal to one of these two candidate levels Q_c^* and Q_e^*.

Corollary 2: *For any* λ *such that* $0 < \lambda < \overline{\lambda}$ *and* s *such that* $s = s_\lambda$*, the output level* Q_{ce} *in* (12) *is monotonically increasing with the demand parameter* λ *.*

Recall that (cf., the characterization derived from Proposition 1 and Corollary 1) Q_e^* is the unique output level that verifies the *ex-post* rate-of-return constraint and is monotonically increasing with the demand parameter λ. This corollary thus confirms that Q_{ce} involves an expansion beyond the *ex-ante* output level Q^* and thus the *ex-post* output level Q_e^* verifies $Q_e^* = Q_{ce}$.

To summarize, we have just shown that if the *ex post* demand expansion coefficient λ is lower than the upper bound value $\overline{\lambda}$, there exists a unique allowed rate of return s_λ with $r < s_\lambda < s_M$ such that: (i) the regulated firm *ex-ante* rationally installs the capital stock $K^* = K(Q_{ce})$ to supply the output Q^*, and (ii) *ex post* , the regulated firm reacts to the expanded demand by increasing its output beyond the output Q^* to attain the level Q_{ce} such that the *ex-post* capital-output combination (K^*, Q_e^*) is cost efficient, that is, $(K^*, Q_e^*) = (K(Q_{ce}), Q_{ce})$.

4. Discussion

4.1 Implications for a natural gas pipeline project

We now adopt a numerical perspective to examine the implications of setting the allowed rate of return at the level s_λ for the pipeline technology studied in Perrotton and Massol (2018), that is, $\alpha = 8/11$ and $\beta = 9/11$. For various conceivable values of the demand price elasticity $1/\varepsilon$ listed in Table 3 (column I), we consider a series of values of the *ex-post* demand expansion parameter λ in the range $0 < \lambda < \overline{\lambda}$ where the upper bound value is $\overline{\lambda} \equiv \left[\frac{\eta}{(1-\varepsilon)\alpha}\right]^{\frac{\eta}{\beta}} \left[\frac{1-\varepsilon}{\beta}\right] - 1$ (see Table 3 – column II). For each of these values, we first numerically evaluate the ratio s_λ / r of the allowed rate of return to the market price of capital such that the *ex-post* expanded output is produced in a cost-efficient manner (i.e., this ratio solves the equation (15)). The ratio s_λ / r is presented in column III.

To explore the implications, we then tabulate two collections of performance ratios that respectively examine the *ex-ante* and *ex-post* situations. The first collection is drawn from Callen et al. (1976) and the second is specific to this paper. For concision, their closed-form expressions are detailed in Appendix B. It should be noted that whatever the ratio under scrutiny, it has a closed-form expression that is invariant with the market prices e and r of the inputs used by the regulated firm.[18]

The first collection is presented in Table 3 – columns IV to VII and focuses on the *ex-ante* situation. The output ratio Q^*/Q_M in column IV compares the output of the regulated firm and that of an unregulated monopoly. The overcapitalization ratio $K^*/K(Q^*)$ in column V compares the amount of capital stock installed by the regulated firm to the amount needed to serve the same output at a minimum long-run cost. The cost ratio $C^*/C(Q^*)$ documents the extra cost generated by that overcapitalization (column VI). In column VII, we report the net social welfare ratio $(W^* - W_M)/(W_a - W_M)$ where W_M (respectively, W_a) is the net social welfare obtained if the *ex-ante* demand is served by an unregulated private monopoly (respectively, by a welfare-maximizing social planner providing zero profit to the firm). This ratio thus compares the gain in net social welfare $(W^* - W_M)$ resulting from the application of the rate-of-return regulation on a private monopoly with the gain in net social welfare $(W_a - W_M)$ that would be obtained by a social planner applying the average cost-pricing rule in a previously monopolistically-controlled industry.

The second collection of indicators in columns VIII to X focuses on the *ex post* situation, once the expanded demand materializes. The output expansion ratio Q_{ce}/Q^* in column VIII assesses the magnitude of the change observed in the firm's output by comparing the firm's *ex-post* and *ex-ante* output levels. In column IX, we examine the price implications and report the price ratio $P_\lambda(Q_{ce})/P(Q^*)$ that relates the *ex-post* price level $P_\lambda(Q_{ce})$ observed when the firm produces the output Q_{ce} to the *ex-ante* price level $P(Q^*)$. Lastly, one may wish to explore the social performance of the regulated sector once the expanded demand

[18] *These ratios are entirely determined by the technological parameters* α *and* β*, the ratio* s_λ/r*, the demand price elasticity* $1/\varepsilon$ *and the ex-post demand expansion parameter* λ*.*

materializes. So, we report in column X the ratio $\left(W_{ce}^{\lambda}-W_{M}^{\lambda}\right)/\left(W_{a}^{\lambda}-W_{M}^{\lambda}\right)$ where W_{ce}^{λ} is the net social welfare attained *ex post* and W_{M}^{λ} (respectively, W_{a}^{λ}) is the net social welfare obtained if the *ex-post* demand is served by an unregulated private monopoly (respectively, by a social planner applying the average-cost-pricing rule).

Table 3. Rate of return, output, cost, price and welfare gain ratios for alternative demand elasticities and demand expansion parameters

[Please insert Table 3 here]

The *ex-ante* ratios in Table 3 indicate that, in the case of a large demand expansion coefficient λ that is close to $\overline{\lambda}$, the allowable rate of return s_{λ} has to be set at a low level (i.e., close to the cost of capital r) to obtain a sufficiently large degree of overcapitalization (cf. column V). Of course, this overcapitalization *ex ante* imposes an extra cost that can be substantial (see column VI). Yet, it should be noted that the output of the regulated firm is considerably larger than that of an unregulated monopoly. Thus, despite the Averch-Johnson distortion, the gain in net social welfare obtained by imposing the RoR regulation to an unregulated monopoly is larger than 70% of the theoretical gain that could be obtained by changing the unregulated monopolist into a social planner applying the average cost-pricing rule.

Regarding the *ex-post* ratios, we note that the occurrence of a larger-than-anticipated demand *ex post* (i.e., $\lambda > 0$) forces the regulated firm to substantially expand its output. With our elasticity and demand expansion parameters, the pipeline output augments by more than 30% (in column VIII, the output ratio is always greater than 1.3 when lambda is strictly positive). This expansion is large enough to systematically yield to a price decline *ex post* (see column IX). It is also interesting to contrast the *ex-post* social welfare ratios in column X with the *ex-ante* values in column VII. By absorbing the overcapitalization, the *ex-post* rise of the pipeline output substantially improves the social performance of the regulated sector.

Two other important observations can also be drawn from Table 3. First, in the case of a small expansion coefficient λ, the ambition to optimally build ahead of demand recommends setting the regulated rate of return s_{λ} to a level significantly higher than the market price of capital r. Indeed, if λ is close to zero, there is a limited need to overcapitalize *ex ante* and the regulated rate of return s_{λ} is set close to the rate of return

obtained by an unregulated private monopoly s_M (because an unregulated monopoly *de facto* uses a cost-efficient combination of inputs). In that case, using the RoR regulation to build ahead of demand would obviously have a detrimental effect on the net social welfare, particularly on the *ex-ante* consumers (i.e., the mega-projects) critically needed to finance the construction of the infrastructure. This last remark echoes the policy discussion in Joskow (1999) who pointed out that the public policy objectives assigned to the regulator (e.g., maximizing the net social welfare, favoring the use of a cost-efficient combination of inputs) can be conflicting goals and a regulator would have to prioritize them.

Second, such a conflict is less pronounced in the case of a larger demand expansion coefficient. Recall that the magnitudes of the *ex-post* effects (i.e., the output increase, the price decline, and the gains in net social welfare) are larger if λ is large and close to the upper bound value $\overline{\lambda}$. In that case, the regulated rate of return is set close to r the market price of capital and the *ex-post* behavior of the regulated firm becomes equivalent to the theoretical benchmark of a social planner applying the average cost-pricing rule. From a regulatory policy perspective, this situation allows us to "kill two birds with one stone" because the traditional goal assigned to the regulation (i.e., augmenting the net social welfare by limiting the exertion of market power by the monopolist operator) is perfectly aligned with the development planning objective to build the infrastructure ahead of demand to minimize the long-run infrastructure cost.

4.2 Conflicting regulatory objectives?

The discussion above suggests that there is a domain over which the two regulatory policy objectives of protecting the society from monopoly prices and "building ahead of demand" are congruent. Yet, an important question still has to be addressed: how large is that domain? To investigate it, we first identify a range of regulated rates of return that provide a high level of net social welfare *ex ante* and then clarify under which conditions these rates allow us to serve the *ex-post* demand in a cost-efficient manner.

<u>The *ex-ante* social welfare</u>

Callen et al. (1976) provide a closed-form expression for the net social welfare W^* obtained in the case of a regulated monopolist serving the *ex-ante* demand schedule (see Table 2). It is a single-variable function

of the regulated rate of return s which, under our assumption, is strictly concave. They also examine how a regulatory agency could set the allowed rate of return so as to maximize the net social welfare given the regulated firm's reaction to that rate. They proved that this socially desirable rate can be larger than the market price of capital and is as follows:

$$s^{opt} = \max\left\{ r, \frac{\eta^2}{\alpha\left[\beta - (1-\alpha)(1-\varepsilon)^2\right]} r \right\}. \tag{16}$$

In a recent note, Perrotton and Massol (2018) incidentally remark that this socially desirable rate of return is bounded[19] – it verifies $s^{opt} \leq \beta r/\alpha$ – and they thus recommend setting the regulated rate of return below $\beta/\alpha = 1.125$ times the market price of capital faced by the pipeline operator. As the social performance of their policy recommendation still has to be evaluated, the following proposition provides a useful result.

Proposition 3: *If the regulated rate of return s is set in the range $r < s \leq \beta r/\alpha$, the net social welfare $W^*(s)$ obtained ex ante is not smaller than $\underline{W^*} = \min\{W^*(r), W^*(\beta r/\alpha)\}$.*

To compare that lower bound with the ideal case of a regulator implementing the socially desirable rate of return s^{opt}, we adopt a numerical perspective. In Table 4, we evaluate, for various conceivable values of the demand price elasticity parameter, two versions of the net social welfare gain ratio obtained *ex ante* that has been presented in section 4.1. The first one is $\left(W^*(s^{opt}) - W_M\right)/(W_a - W_M)$ and measures the social performance of the regulated sector when the socially desirable rate of return s^{opt} is implemented. The second one is $\left(\underline{W^*} - W_M\right)/(W_a - W_M)$ and provides a lower bound for the ratio $\left(W^*(s) - W_M\right)/(W_a - W_M)$ obtained when the regulator arbitrarily sets the rate of return s in the range $r < s \leq \beta r/\alpha$.

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[19] *Recall that $0 < \alpha < \beta < 1$. For any ε in the assumed domain $(1-\beta, 1)$, the gradient of $f : \varepsilon \mapsto \left[\beta - (1-\alpha)(1-\varepsilon)\right]^2 \Big/ \left[\alpha\left(\beta - (1-\alpha)(1-\varepsilon)^2\right)\right]$ is positive, which indicates that f is a smooth and monotonically increasing function. For any ε in $(1-\beta, 1)$, we thus have $f(\varepsilon) < f(1)$ that is: $f(\varepsilon) < \beta/\alpha$.*

Table 4. Welfare gain ratios for alternative demand elasticity parameters

[Please insert Table 4 here]

The results presented in Table 4 indicate that the two performance ratios are very close. Hence, arbitrarily setting the regulated rate of return in the range $r < s \leq \beta r/\alpha$ provides a high level of net social welfare *ex ante* and fulfills the public policy goal to prevent the exertion of market power in the pipeline sector.

<u>Preserving the net social welfare obtained *ex ante* while building ahead of demand</u>

We shall now explore under which conditions the regulated rate of return s_λ, that allows us to serve the *ex-post* demand in a cost efficient manner, is also lower than the threshold $\beta r/\alpha$.

Recall that in section 3.4, Proposition 2 states that there is a one-to-one mapping between the *ex-post* demand expansion parameter λ in the range $0 < \lambda < \overline{\lambda}$ and the regulated rate of return s_λ in the range $r < s_\lambda < s^M$. The following corollary builds on that result to identify a range of values for the demand expansion parameter λ such that the rate of return s_λ is not harmful for the net social welfare obtained *ex ante* while allowing the desired overcapitalization needed for *ex-post* cost efficiency.

Proposition 4: *For any* λ *such that* $\underline{\lambda} \leq \lambda < \overline{\lambda}$ *where* $\underline{\lambda} \equiv \frac{(1-\varepsilon)}{\beta}[\beta - \alpha + 1]\left(\frac{\eta}{(1-\varepsilon)\beta}\right)^{\eta/\beta} - 1$, *there exists a unique allowed rate of return* s_λ *in the interval* $\left(r, \frac{\beta}{\alpha} r\right]$ *such that the condition (15) for the ex-ante installation of the capital stock needed to serve the ex-post demand in a cost-efficient manner is verified.*

Hence, if the demand expansion coefficient λ verifies $\underline{\lambda} \leq \lambda < \overline{\lambda}$, the regulator can set the allowable rate of return at a level s_λ that is not greater than the upper bound $\beta r/\alpha$ and thus jointly fulfill the two public policy objectives of preserving a high level of net social welfare *ex ante* and inducing the regulated firm to build ahead of demand by installing the targeted amount of extra-capital stock.

To gain insights into the width of the interval $\left[\underline{\lambda},\overline{\lambda}\right)$ for a natural gas pipeline, Table 5 reports the values of the upper and lower bounds for a series of conceivable values of the demand price elasticity $1/\varepsilon$. Table 5 also reports the output expansion ratio Q_{ce}/Q^{*} that compares the firm's *ex-post* and *ex-ante* output levels obtained under the two cases $\lambda = \underline{\lambda}$ and $\lambda = \overline{\lambda}$.

Table 5. The range of demand expansion (or output expansion) such that it is possible to build ahead of demand while preserving the *ex-ante* net social welfare

[Please insert Table 5 here]

The values detailed in Table 5 indicate that the interval $\left[\underline{\lambda},\overline{\lambda}\right)$ (and the associated range of output expansion ratios Q_{ce}/Q^{*}) is narrow. This finding reveals that the domain over which RoR regulation can be used to optimally build ahead of demand while preserving a high level of net social welfare *ex ante* is quite limited.[20] These results also support an important policy recommendation for the regulation of the natural gas pipeline sector in a developing country: as RoR regulation is unlikely to "kill two birds with one stone," it is very important to decide which of the two public policy objectives should be assigned a high priority.

5. Conclusions and policy implications

Developing countries trying to develop natural gas resources through pipeline infrastructure face a number of challenges. On the one hand, they need to impose a clear and manageable regulatory framework to the future pipeline operator, such as the long-advocated rate-of-return regulation. But this framework presents its own flaws, as it is suspected of generating over-investment through the Averch-Johnson effect. On the other hand, they must provide an infrastructure at times where initial demand is almost inexistent and prepare for future demand growth. While the irreversibility and increasing returns of pipeline investments,

[20] *For example, in Mozambique, the development scenarios in ICF (2012) mention the ex-ante installation of four "mega-projects": two power plants (one in Pemba and one in Nacala), a fertilizer plant and a methanol processing one representing an aggregated annual consumption of natural gas of 2×9.5+15.6+18 = 52.6 Bcf. The ex-post scenarios envision the possible additional installation of one to three power plants plus possibly a large "Gas-to-Liquid" plant representing a rise of the pipeline throughout that ranges from +18% to +435%. That said, given the lumpy nature of these gas-based industrial projects, and for reasonable values of the demand elasticity, it is very unlikely that the associated demand expansion parameters* λ *will verify* $\underline{\lambda} \leq \lambda < \overline{\lambda}$.

derived from their engineering characteristics, advise them to build ahead of demand, this would likely collide with the more conservative approach of foreign investors.

We show in this paper that economic analysis can help to a certain extent to address these challenges simultaneously. Using classical rate-of-return regulation models, we examine the design choices of a regulated firm based on *ex-ante* conservative demand estimates, and extend the literature by characterizing its operating decisions once it reacts to a larger than expected *ex-post* demand. We then prove that a regulator can choose the allowed rate-of-return *ex ante* so as to induce the firm to build ahead of demand. This is a crucial finding, as it guarantees an efficient *ex-post* operation and a reduction in the Averch-Johnson distortions.

This strategy has several limitations, though. It can only be applied for a demand growth under an identified threshold and may impact the initial welfare in the case of large allowed rates of return. We also show using numerical data that the range of demand growth ratios for which *ex-post* welfare can be improved and initial adverse effects kept limited is so narrow that the regulators will likely have to prioritize one goal over the other in practice.

The analysis developed here provides important novel insights for development planners and regulators involved with pipeline infrastructure projects in developing countries. Given the significant investments and large economic potential at stake in such projects, it can greatly contribute to addressing the contradictory challenges they face, as shown for the case of the Rovuma fields in Mozambique. It also demonstrates that these are infrastructure projects for which it is crucial to clearly prioritize policy goals to achieve the desired outcomes.

The analysis in this paper is based on a simplified representation and, as in any simple model, some extensions can be proposed to further refine and enrich the analysis. At least three lines of possible future extensions can be envisioned. First, it could be interesting to investigate the effects of vertical integration between gas retailing and pipeline transportation activities on the deployment of such infrastructures (i.e., to examine whether revenues from gas sales could partly finance the construction of the pipeline ahead of demand). Second, the possibility for the pipeline operator to implement discriminatory pricing schemes

between the different users of the infrastructure could also represent an appealing extension. Lastly, it is important to keep in mind that the present analysis solely focuses on RoR regulation and thus overlooks alternative institutional organizations (e.g., price-cap regulation, reverse capacity auctions). Analyzing the ability of these schemes to: (i) attract an adequate level of investment *ex ante*, while (ii) preserving society form monopoly prices and (iii) meeting the specific challenges faced in developing nations (e.g., the shortage of skilled capital in local regulatory authorities) could usefully inform the regulatory policy debates observed in developing countries.

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Appendix A – Mathematical proofs

Proof of Proposition 1: *Ex post, the capital stock is fixed and equals* K^*. *The regulated firm's profit is given by the single variable function* $\Pi : Q \mapsto (1+\lambda)P(Q)Q - rK^* - eE(Q,K^*)$ *that is twice-differentiable, strictly concave, and verifies* $\Pi(0) = -rK^*$ *and* $\lim_{Q\to+\infty} \Pi(Q) = -\infty$. *We let* M *denote the unique profit-maximizing output. Here, the profit function* Π *is monotonically increasing (respectively, decreasing) on the left interval* $[0,M]$ *(respectively, the right interval* $[M,+\infty)$*) and there is a one-to-one correspondence between the left interval* $[0,M]$ *(respectively, the right interval* $[M,+\infty)$*) and the image* $[\Pi(0),\Pi(M)]$ *(respectively, the interval* $(-\infty,\Pi(M)]$*).*

Recall that we are looking for an output level Q *such that the ex-post rate-of-return constraint* (9) *is verified. The condition* (9) *is logically equivalent to* $\Pi(Q) = (s-r)K^*$.

We first focus on the right interval $[M,+\infty)$ *and are going to prove that the image interval* $(-\infty,\Pi(M)]$ *contains the value* $(s-r)K^*$. *Notice that the output level* Q^* *verifies* (5) *and that*

$\Pi(Q^*) = \lambda P(Q^*)Q^* + (s-r)K^*$. *As* $\lambda > 0$ *and* $Q^* > 0$, *we obtain* $\Pi(Q^*) > (s-r)K^*$. *Using the definition of a maximum:* $\Pi(M) \geq \Pi(Q^*)$. *So,* $\Pi(M) > (s-r)K^*$ *which proves that the open interval* $(-\infty, \Pi(M))$ *contains* $(s-r)K^*$. *Hence, there exists a unique output level* Q_e^* *in* $(M, +\infty)$ *such that* $\Pi(Q_e^*) = (s-r)K^*$ *and the condition* (9) *holds.*

Then, we examine the left interval $[0, M]$. *As* $s > r$, *we have* $(s-r)K^* > 0$ *and thus* $(s-r)K^* > \Pi(0)$. *As we have already shown that* $\Pi(M) > (s-r)K^*$, *we can now affirm that the open interval* $(\Pi(0), \Pi(M))$ *also contains* $(s-r)K^*$. *So, there also exists a unique output level* Q_c^* *in* $(0, M)$ *such that* $\Pi(Q_c^*) = (s-r)K^*$ *and the constraint* (9) *is verified.*

We have just shown that there exists two solutions Q_c^* *and* Q_e^* *that verify* $Q_c^* < M < Q_e^*$. *Now, recall that the pair* Q^* *and* K^* *verifies* (6). *As* $s > r$, *the firm's ex-post marginal profit evaluated at* Q^* *thus verifies* $\Pi'(Q^*) = \lambda\left[P'(Q^*)Q^* + P(Q^*)\right]$ *which is positive because:* $\lambda > 0$; $P'(Q^*)Q^* + P(Q^*) = eE_Q(Q^*, K^*)$ *(cf., equation (6)) and* $E_Q(Q, K) > 0$. *As* $\Pi'(Q^*) > 0$, *the marginal profit function is locally monotonically increasing. Because of the strict concavity of the profit function, it means that* $Q^* < M$ *and thus* $Q^* < Q_e^*$. *Recall that we have shown above that* $\Pi(Q^*) > (s-r)K^*$. *As the profit function is monotonically increasing on the interval* $[0, M]$, *the condition* $Q_c^* < Q^*$ *also holds. So, the two solutions verify* $Q_c^* < Q^* < Q_e^*$. *Q.E.D.*

Proof of Corollary 1: *Recalling that* Q_e^* *(respectively,* Q_c^* *) verifies the ex-post regulatory constraint* (9) *which is logically equivalent to* $(1+\lambda)P(Q)Q - eE(Q, K^*) = (s-r)K^*$, *the implicit function theorem can be invoked to assess the sign of* $dQ_e^*/d\lambda$ *(respectively,* $dQ_c^*/d\lambda$ *). As:* $\partial\left[(1+\lambda)P(Q)Q - eE(Q, K^*) - (s-r)K^*\right]/\partial\lambda$ *evaluated at* $Q = Q_e^*$ *(respectively,* $Q = Q_c^*$ *) equals* $P(Q_e^*)Q_e^*$ *(respectively,* $P(Q_c^*)Q_c^*$ *) which is positive, and* $\partial\left[(1+\lambda)P(Q)Q - eE(Q, K^*) - (s-r)K^*\right]/\partial Q$ *evaluated at* $Q = Q_e^*$ *(respectively,* $Q = Q_c^*$ *) equals the ex-post marginal profit* $\Pi'(Q_e^*)$ *(respectively,* $\Pi'(Q_c^*)$ *)*

introduced in the preceding proof which is negative (respectively positive), the implicit function theorem reveals that $dQ_e^* / d\lambda > 0$ *(respectively,* $dQ_c^* / d\lambda < 0$ *).* *Q.E.D.*

Proof of Proposition 2: *Recall that the condition* $K^* = K_{ce}$ *is equivalent to:*

$$\frac{(1-\varepsilon)}{\beta}\left[\left(\frac{s}{r}-1\right)\alpha+1\right]\left(\frac{r}{s}\frac{\eta}{(1-\varepsilon)\alpha}\right)^{\eta/\beta}-1=\lambda. \tag{A.1}$$

We let $x \in (1, s_M/r)$ *denote the ratio* s/r *and let* $f : x \mapsto \left[(x-1)\alpha+1\right]x^{-\eta/\beta}$. *We are going to prove that this smooth univariate function is a monotonically decreasing one. We let:* $v : x \mapsto \beta\left[(x-1)\alpha+1\right]x$. *Remarking that* $v(x) > 0$ *and* $f(x) > 0$ *for any* $x > 1$, *it is clear that the sign of* $v(x)\frac{f'(x)}{f(x)} = -\eta\left[(x-1)\alpha+1\right]+\beta\alpha x$ *is identical to that of* $f'(x)$ *the gradient of* f *w.r.t.* x *evaluated at* x. *Recalling that* $\eta \equiv \beta-(1-\varepsilon)(1-\alpha)$ *and rearranging, we obtain* $v(x)\frac{f'(x)}{f(x)} = (1-\alpha)\left[(1-\varepsilon)\alpha x-\eta\right]$. *As* $(1-\varepsilon)\alpha > 0$ *and* $(1-\alpha) > 0$, *the expression* $(1-\alpha)\left[(1-\varepsilon)\alpha x-\eta\right]$ *which is a linear function of the variable* x *has a positive slope coefficient and is thus a monotonically increasing function. So,* $v(x)\frac{f'(x)}{f(x)} < v\left(\frac{s_M}{r}\right)\frac{f'(s_M/r)}{f(s_M/r)}$ *for any* $x \in (1, s_M/r)$. *As* $s_M/r = \left[\beta/(1-\varepsilon)-(1-\alpha)\right]/\alpha$, *we have* $v\left(\frac{s_M}{r}\right)\frac{f'(s_M/r)}{f(s_M/r)} = 0$ *which proves that* $f'(x) < 0$ *for any* $x \in (1, s_M/r)$. *We have just shown that the smooth univariate function* f *is monotonically decreasing which indicates that the smooth univariate function* $h : s \mapsto \left[(1-\varepsilon)/\beta\right]\left[\eta/(\alpha(1-\varepsilon))\right]^{\eta/\beta} f(s/r)-1$ *is also monotonically decreasing. Hence,* h *is a one-to-one mapping from the open interval* (r, s_M) *to the image interval* $(h(s_M), h(r))$ *where* $h(s_M) = 0$ *and* $h(r) = \left[\frac{\eta}{(1-\varepsilon)\alpha}\right]^{\frac{\eta}{\beta}}\left[\frac{1-\varepsilon}{\beta}\right]-1$ *that is* $\overline{\lambda}$.

As the function h is invertible, we let $g:\lambda \mapsto h^{-1}(\lambda)$ denote its inverse. By construction, g is also a one-to-one mapping from the open interval $\left(0,\overline{\lambda}\right)$ to the interval (r,s_M) and the value of its derivative for any $\lambda \in \left(0,\overline{\lambda}\right)$ is $g'(\lambda)=1/h'(s)$ where s is the unique return in $\left(r,s^M\right)$ such that $s=g(\lambda)$ (cf., the inverse function theorem). As the sign of $h'(s)$ equals the one of $f'(s/r)$ and it has been shown above that the latter is negative for any $s\in\left(r,s^M\right)$, we thus have $g'(\lambda)<0$ which indicates that g is a monotonically decreasing function of the demand parameter λ. *Q.E.D.*

Proof of Corollary 2: *We assume that s is set at the level s_λ mentioned in Proposition 2. Inserting first s_λ in the closed-form expression of the output level Q_{ce} detailed in equation (12) and then remarking that $A(\alpha/r)^{\alpha}\left((1-\alpha)/e\right)^{1-\alpha}>0$, that $\lambda \geq 0$ and that $s_\lambda \geq r$, the sign of the gradient of Q_{ce} with respect to the demand parameter λ is:*

$$sign\left(\frac{dQ_{ce}}{d\lambda}\right)=sign\left(\frac{1}{(1+\lambda)\gamma}-\frac{1}{\gamma}.\frac{ds_\lambda}{d\lambda}.\frac{1}{\left(\frac{s_\lambda}{r}-1\right)\alpha+1}\right) \tag{A.2}$$

Recall that γ is positive (as by assumption $\varepsilon>1-\beta$), that $s_\lambda \geq r$ and that Proposition 2 indicates that $\left(ds_\lambda/d\lambda\right)<0$. So, $\left(dQ_{ce}/d\lambda\right)>0$ and the cost-efficient output level Q_{ce} is monotonically increasing with the demand parameter λ. *Q.E.D.*

Proof of Proposition 3: *If the regulator sets the rate of return at a level $s>r$, the net social welfare is given by the smooth univariate function $W^*:s\mapsto \frac{A}{1-\varepsilon}\left[Q^*(s)\right]^{1-\varepsilon}-rK^*(s)-eE\left(Q^*(s),K^*(s)\right)$, where $Q^*(s)$ and $K^*(s)$ are the regulated firm's optimal decisions. Inserting the value of K^* in Table 2 (see equation III) into $E\left(Q^*(s),K^*(s)\right)$, the total cost of the energy input is $\frac{e}{\delta}\left[Q^*(s)\right]^{1-\varepsilon}$. Using the RoR constraint (5), the total capital cost $rK^*(s)$ is: $\frac{r}{s}\left[P\left(Q^*(s)\right)Q^*(s)-eE\left(Q^*(s),K^*(s)\right)\right]=\frac{r}{s}\left(A-\frac{e}{\delta}\right)\left[Q^*(s)\right]^{1-\varepsilon}$. Simplifying, we have the*

smooth univariate function: $W^*(s)=\frac{A}{\beta}\left(\frac{\beta-(1-\alpha)(1-\varepsilon)^2}{1-\varepsilon}-\frac{r}{s}\eta\right)\left[Q^*(s)\right]^{1-\varepsilon}$ *where* $Q^*(s)$ *is detailed in Table 2 (see equation II). The gradient of* W^* *w.r.t.* s *is:* $\frac{dW^*}{ds}(s)=\frac{A}{\beta s\gamma}\left[\frac{r}{s}\eta^2-\alpha\left[\beta-(1-\alpha)(1-\varepsilon)^2\right]\right]\left[Q^*(s)\right]^{1-\varepsilon}$ *which is positive if* $s<f(\varepsilon)r$, *equal to zero if* $s=f(\varepsilon)r$ *and negative if* $s>f(\varepsilon)r$, *where* $f:\varepsilon\mapsto\eta^2\Big/\left[\alpha\left(\beta-(1-\alpha)(1-\varepsilon)^2\right)\right]$ *is smooth. Given our assumptions, the gradient of* f *is positive for any* ε *in* $(1-\beta,1)$. *So,* f *is a one-to-one mapping between* $\varepsilon\in(1-\beta,1)$ *and* $f(\varepsilon)\in\left(f(1-\beta),f(1)\right)$. *Recall that* $0<\alpha<\beta<1$ *and thus* $f(1)=\beta/\alpha>1$ *and* $f(1-\beta)=\left[1\Big/\left(\frac{1-\beta}{\alpha\beta}+1\right)\right]<1$. *So, there exists a unique* ε^* *in* $(1-\beta,1)$ *such that* $f(\varepsilon^*)=1$. *Two cases have to be discussed depending on whether the elasticity parameter* ε *verifies* $1-\beta<\varepsilon\le\varepsilon^*$ *or* $\varepsilon^*<\varepsilon<1$.

<u>Case 1:</u> $\varepsilon\in\left(1-\beta,\varepsilon^*\right]$ *and thus* $f(\varepsilon)\le 1$. *For any* $s\in(r,\beta r/\alpha)$, *the condition* $s>f(\varepsilon)r$ *is thus verified. Thus, the gradient of* W^* *w.r.t.* s *is negative and* W^* *is a monotonically decreasing function on the interval* $s\in(r,\beta r/\alpha)$. *So,* $W^*(s)>W^*(\beta r/\alpha)$ *and thus* $W^*(s)>\mathrm{Min}\left\{W^*(r),W^*(\beta r/\alpha)\right\}$ *holds.*

<u>Case 2:</u> $\varepsilon\in\left(\varepsilon^*,1\right)$ *and thus* $1<f(\varepsilon)<f(1)$ *where* $f(1)=\beta/\alpha$. *Thus, for any* $s\in\left(r,f(\varepsilon)r\right)$ *(respectively,* $s\in\left(f(\varepsilon)r,\beta r/\alpha\right)$*), the function* W^* *is smooth and monotonically increasing (respectively, decreasing) and the condition* $W^*(s)>W^*(r)$ *(respectively,* $W^*(s)>W^*(\beta r/\alpha)$*) holds. So, the condition* $W^*(s)>\mathrm{Min}\left\{W^*(r),W^*(\beta r/\alpha)\right\}$ *holds.* *Q.E.D.*

Proof of Proposition 4: *We simply have to highlight that* $\underline{\lambda}$ *is the value of the demand expansion parameter obtained by substituting* $s_\lambda=\beta r/\alpha$ *into equation (15). As it has been assumed that* $\beta>\alpha>0$, *the rate* $s_\lambda=\beta r/\alpha$ *is in the interval* $\left(r,s^M\right)$. *Thus, using the one-to-one mapping highlighted in the Proof of Proposition 2, we claim: (i) that setting* $s_\lambda=\beta r/\alpha$ *is the unique allowed rate of return such that the*

equation (15) is verified when $\lambda = \underline{\lambda}$*, and (ii) that* $\underline{\lambda}$ *belongs to the interval* $\left(0, \overline{\lambda}\right)$*. As Proposition 2 also indicates that* s_λ *is monotonically decreasing with* λ*, we can conclude that for any* λ *in the interval* $\left[\underline{\lambda}, \overline{\lambda}\right)$*, there exists a rate of return* s_λ *the interval* $\left(r, \frac{\beta}{\alpha} r\right]$ *such that the condition (15) is verified.* *Q.E.D.*

Appendix B – Ex-post performance ratios

In this appendix, we derive the closed-form expressions of the three ratios used in section 4 to assess the *ex-post* performance of the regulation (i.e., once the demand expansion materializes). These three ratios respectively document the output expansion, the price variation, and the impact on the net social welfare. Hereafter, it is assumed that the allowed rate of return is set at the level s_λ indicated in Proposition 2.

Output expansion ratio

We consider the output expansion ratio Q_{ce}/Q^* that compares the regulated firm's *ex-post* output level Q_{ce} and the *ex-ante* one Q^* to document the magnitude of the change in the firm's production plan. To rapidly obtain a closed-form expression for that ratio, we use the following reformulation where the output level Q_M chosen by an unregulated monopoly facing the *ex-ante* inverse demand (4) is introduced:

$$\frac{Q_{ce}}{Q^*} = \frac{Q_{ce}}{Q_M} \times \frac{Q_M}{Q^*}. \tag{B.1}$$

A closed-form expression of the ratio Q^*/Q_M has been presented above (cf., equation VI in Table 1). Using that expression, the value of Q_{ce} in equation (12), those of Q_M presented in Table 1 (equation IV) and simplifying, we can rewrite the output expansion ratio as follows:

$$\frac{Q_{ce}}{Q^*} = \left[\frac{(1+\lambda)\beta}{(1-\varepsilon)\left[\left(\frac{s_\lambda}{r} - 1\right)\alpha + 1\right]}\right]^{1/\gamma} \times \left(\frac{s_\lambda}{r}\frac{(1-\varepsilon)\alpha}{\eta}\right)^{\alpha/\gamma}. \tag{B.2}$$

Price ratio

We now examine the price ratio $P_\lambda(Q_{ce})/P(Q^*)$ that provides a rapid comparison between the *ex-post* price level $P_\lambda(Q_{ce})$ observed when the regulated firm produces the output Q_{ce} and the *ex-ante* price level $P(Q^*)$. Using the definitions of the inverse demand functions in equations (4) and (8), we obtain:

$$\frac{P_\lambda(Q_{ce})}{P(Q^*)} = (1+\lambda)\left(\frac{Q_{ce}}{Q^*}\right)^{-\varepsilon} \tag{B.3}$$

where Q_{ce}/Q^* is the output expansion ratio in (B.2).

<u>Net social welfare</u>

To document the implications for the net social welfare, we consider the ratio $(W_{ce}^\lambda - W_M^\lambda)/(W_a^\lambda - W_M^\lambda)$ where W_{ce}^λ is the net social welfare attained *ex post* and W_M^λ (respectively, W_a^λ) is the net social welfare that would have been obtained if the *ex-post* demand had been served by an unregulated private monopoly (respectively, by a social planner applying the average-cost-pricing rule) that could freely decide the optimal output-capital pair needed to serve the *ex-post* demand.

To begin with, we evaluate the net social welfares W_M^λ and W_a^λ. Substituting the *ex-post* inverse demand function (8) in the optimization program stated in Table 1 yields the optimal output level Q_M^λ (respectively, Q_a^λ) decided by the unregulated private monopoly (respectively, the social planner):

$$Q_M^\lambda = \left[\frac{(1+\lambda)A(1-\varepsilon)}{\beta}\left(\frac{\alpha}{r}\right)^\alpha\left(\frac{1-\alpha}{e}\right)^{1-\alpha}\right]^{\frac{1}{\gamma}} \tag{B.4}$$

$$Q_a^\lambda = \left[(1+\lambda)A\left(\frac{\alpha}{r}\right)^\alpha\left(\frac{1-\alpha}{e}\right)^{1-\alpha}\right]^{\frac{1}{\gamma}} \tag{B.5}$$

Moreover, with the constant elasticity demand schedule (8), the net social welfare associated with Q units of output can be written: $W^\lambda = \left[(1+\lambda)A/(1-\varepsilon)\right]Q^{1-\varepsilon} - C(Q)$ where $C(Q)$ is the total cost indicated in equation (3).

As Q_a^λ is the output such that price equals the average cost: $(1+\lambda)A\left(Q_a^\lambda\right)^{1-\varepsilon} = C\left(Q_a^\lambda\right)$, the net social welfare obtained under a social planner applying the average-cost-pricing rule is:

$$W_a^\lambda = P\left(Q_a^\lambda\right)Q_a^\lambda\left[\varepsilon/(1-\varepsilon)\right]. \tag{B.6}$$

Remarking that $Q_M^\lambda = \sqrt[\gamma]{((1-\varepsilon)/\beta)}Q_a^\lambda$ and using the relation $(1+\lambda)A\left(Q_a^\lambda\right)^{1-\varepsilon} = C\left(Q_a^\lambda\right)$, the net social welfare obtained in the case of a monopoly is:

$$W_M^\lambda = P\left(Q_a^\lambda\right)Q_a^\lambda.\left(\frac{1}{1-\varepsilon}\left(\frac{1-\varepsilon}{\beta}\right)^{\frac{1-\varepsilon}{\gamma}} - \left(\frac{1-\varepsilon}{\beta}\right)^{\frac{\beta}{\gamma}}\right). \tag{B.7}$$

Similarly, one can observe that the *ex-post* cost-efficient output of the regulated firm is directly proportional to the output chosen by the social planner applying the average cost-pricing rule as:

$$Q_{ce} = \left[\left(\frac{s_\lambda}{r}-1\right)\alpha+1\right]^{-1/\gamma} Q_a^\lambda. \tag{B.8}$$

Hence, one can also use the relation $(1+\lambda)A\left(Q_a^\lambda\right)^{1-\varepsilon} = C\left(Q_a^\lambda\right)$ to write the net social welfare obtained by the regulated monopoly that increases its output to cope with the augmented demand:

$$W_{ce}^\lambda = P\left(Q_a^\lambda\right)Q_a^\lambda.\left(\frac{1}{1-\varepsilon}\left[\left(\frac{s_\lambda}{r}-1\right)\alpha+1\right]^{\frac{\varepsilon-1}{\gamma}} - \left[\left(\frac{s_\lambda}{r}-1\right)\alpha+1\right]^{\frac{-\beta}{\gamma}}\right). \tag{B.9}$$

Using (B.9), (B.7), (B.6) and simplifying, one can readily obtain a simple expression for the ratio $\left(W_{ce}^\lambda - W_M^\lambda\right)/\left(W_a^\lambda - W_M^\lambda\right)$ that depends solely on the technological parameters (i.e., α and β), the demand price elasticity and the ratio s_λ/r of the allowed rate of return to the market price of capital.

Table 1. The cases of a profit-maximizing, unregulated monopoly and a welfare-maximizing social planner providing zero profit to the firm

	The unregulated monopoly	The welfare-maximizing planner that provides zero-profits to the firm
Optimization program	$\underset{K,Q}{\text{Max}}\ \Pi_M(Q) = P(Q)Q - rK - eE(Q,K)$ (I)	$\underset{K,Q}{\text{Max}}\ W(Q) = \int_0^Q P(q)\,dq - rK - eE(Q,K)$ (II) s.t. $P(Q)Q - rK - eE(Q,K) = 0$ (III)
Solution:		
Output	$Q_M = \left[\frac{A(1-\varepsilon)}{\beta}\left(\frac{\alpha}{r}\right)^{\alpha}\left(\frac{1-\alpha}{e}\right)^{1-\alpha}\right]^{\frac{1}{\gamma}}$ (IV)	$Q_a = \left[A\left(\frac{\alpha}{r}\right)^{\alpha}\left(\frac{1-\alpha}{e}\right)^{1-\alpha}\right]^{\frac{1}{\gamma}}$ (V)
Capital	$K_M = \left(\frac{e\alpha}{r(1-\alpha)}\right)^{1-\alpha}(Q_M)^{\beta}$ (VI)	$K_a = \left(\frac{e\alpha}{r(1-\alpha)}\right)^{1-\alpha}(Q_a)^{\beta}$ (VII)

Note: The objective function (I) is the firm's profit, i.e.: the difference between the total revenue $P(Q)Q$ *and the sum of the capital cost* rK *and the energy cost* $eE(Q,K)$. *The objective function (II) is the net social welfare defined as the sum of the consumer surplus* $\int_0^Q P(q)\,dq - P(Q)Q$ *and the producer's surplus* $P(Q)Q - rK - eE(Q,K)$. *The constraint (III) states that the firm is compelled to obtain zero economic profit.*

Table 2. The *ex-ante* behavior of the regulated firm

	The regulated monopoly	
Panel 1: **Optimization program if** $s \leq s_M$	$\underset{K,Q}{\text{Max}} \quad \Pi(Q) = P(Q)Q - r\,K - e\,E(Q,K)$ $\text{s.t.} \quad P(Q)Q - e\,E(Q,K) = s\,K$ $K \geq 0 \;,\; Q \geq 0\,.$	(I)
Panel 2: **Solution if** $r \leq s \leq s_M$		
Output	$Q^* = \left[\dfrac{A\delta - e}{s\delta^{1/\alpha}}\right]^{\alpha/\gamma}$	(II)
Capital	$K^* = \delta^{(1-\alpha)/\alpha} Q^{*\eta/\alpha}$	(III)
Panel 3: **Implications**		
Cost	$C^* = rK^* + eE\left(Q^*, K^*\right) = r\delta^{(1-\alpha)/\alpha} Q^{*\eta/\alpha} + \dfrac{e}{\delta} Q^{*1-\varepsilon}$	(IV)
Net Social Welfare	$W^* = \int_0^{Q^*} P(q)\,dq - C^* = \dfrac{1}{1-\varepsilon} P\left(Q^*\right)Q^* - C^*$	(V)
Output ratio	$\dfrac{Q^*}{Q_M} = \left(\dfrac{r}{s}\dfrac{\eta}{(1-\varepsilon)\alpha}\right)^{\alpha/\gamma}$	(VI)
Overcapitalization ratio	$\dfrac{K^*}{K\left(Q^*\right)} = \left(\dfrac{r}{s}\dfrac{\eta}{(1-\varepsilon)\alpha}\right)^{1-\alpha}$	(VII)
Cost ratio	$\dfrac{C^*}{C\left(Q^*\right)} = (1-\alpha)\left[\dfrac{s}{r}\dfrac{(1-\varepsilon)\alpha}{\eta}\right]^{\alpha} + \alpha\left[\dfrac{r}{s}\dfrac{\eta}{(1-\varepsilon)\alpha}\right]^{1-\alpha}$	(VIII)

Note: If $s = r$ *, the pair* $\left(K^*, Q^*\right)$ *is not the unique solution to the optimization program (Klevorick, 1971).*

Table 3. Rate of return, output, cost, price and welfare gain ratios for alternative demand elasticities and demand expansion parameters

Column # I	II	III	IV	V	VI	VII	VIII	IX	X
			***Ex-ante* ratios**				***Ex-post* ratios**		
$\frac{1}{\varepsilon}$	λ	$\frac{s_\lambda}{r}$	$\frac{Q^*}{Q_M}$	$\frac{K^*}{K(Q^*)}$	$\frac{C^*}{C(Q^*)}$	$\frac{W^*-W_M}{W_a-W_M}$	$\frac{Q_{ce}}{Q^*}$	$\frac{P_\lambda(Q_{ce})}{P(Q^*)}$	$\frac{W_{ce}^\lambda-W_M^\lambda}{W_a^\lambda-W_M^\lambda}$
1.001	0.000	1,125.750	1.000	1.000	1.000	0.000	1.000	1.000	0.000
	0.050	7.195	89.736	3.967	2.892	0.604	5.388	0.195	0.824
	0.100	3.660	163.778	4.770	3.474	0.669	6.750	0.163	0.908
	0.150	2.455	233.631	5.319	3.871	0.699	7.711	0.149	0.949
	0.200	1.848	300.908	5.748	4.183	0.716	8.478	0.142	0.971
	0.300	1.236	430.329	6.414	4.666	0.728	9.693	0.134	0.993
	$\bar{\lambda}$ =0.371	1.000	519.647	6.795	4.944	0.728	10.403	0.132	1.000
1.150	0.000	8.250	1.000	1.000	1.000	0.000	1.000	1.000	0.000
	0.050	2.355	3.764	1.408	1.133	0.554	1.519	0.730	0.724
	0.100	1.594	5.687	1.566	1.221	0.673	1.730	0.683	0.885
	0.150	1.225	7.518	1.682	1.292	0.720	1.889	0.662	0.962
	$\bar{\lambda}$ =0.200	1.000	9.314	1.778	1.352	0.729	2.021	0.651	1.000
1.300	0.000	4.500	1.000	1.000	1.000	0.000	1.000	1.000	0.000
	0.050	1.632	3.511	1.319	1.089	0.616	1.402	0.810	0.783
	0.100	1.163	5.338	1.446	1.154	0.732	1.570	0.778	0.951
	$\bar{\lambda}$ =0.131	1.000	6.438	1.507	1.187	0.750	1.651	0.769	1.000
1.500	0.000	3.000	1.000	1.000	1.000	0.000	1.000	1.000	0.000
	0.050	1.235	3.787	1.274	1.070	0.707	1.344	0.862	0.876
	$\bar{\lambda}$ =0.082	1.000	5.196	1.349	1.104	0.781	1.442	0.847	1.000
1.700	0.000	2.357	1.000	1.000	1.000	0.000	1.000	1.000	0.000
	0.050	1.033	4.374	1.252	1.060	0.797	1.316	0.893	0.976
	$\bar{\lambda}$ =0.055	1.000	4.638	1.263	1.065	0.811	1.331	0.891	1.000

Table 4. Welfare gain ratios for alternative demand elasticity parameters

$\frac{1}{\varepsilon}$	$\frac{s^{opt}}{r}$	$\frac{W^*\left(s^{opt}\right)-W_M}{W_a-W_M}$	$\frac{\underline{W}^*-W_M}{W_a-W_M}$
1.001	1.124	0.729	0.728
1.150	1.035	0.729	0.727
1.300	1.000	0.750	0.738
1.500	1.000	0.781	0.748
1.700	1.000	0.811	0.749

Table 5. The range of demand expansion (or output expansion) such that it is possible to build ahead of demand while preserving the *ex-ante* net social welfare

$\frac{1}{\varepsilon}$	$\underline{\lambda}$	$\overline{\lambda}$	$\frac{Q_{ce}}{Q^*}\left(\underline{\lambda}\right)$	$\frac{Q_{ce}}{Q^*}\left(\overline{\lambda}\right)$
1.001	0.330	0.371	10.002	10.403
1.150	0.170	0.200	1.943	2.021
1.300	0.106	0.131	1.587	1.651
1.500	0.063	0.082	1.387	1.442
1.700	0.039	0.055	1.280	1.331

Supplementary Document to "Rate-of-return regulation to unlock natural gas pipeline deployment: the case of Mozambique"

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In this supplementary document, we present a concise derivation of the production function used in the paper. That presentation is based on Perrotton, F., Massol, O. (2018). "The technology and cost structure of a natural gas pipeline: Insights for costs and rate-of-return regulation" *Utilities Policy*, 53: 32–37).

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A simplified representation of the technology of a gas pipeline

We consider a simple point-to-point pipeline infrastructure that consists of a compressor station injecting a pressurized flow of natural gas Q into a pipeline to transport it across a given distance l.

Following Chenery (1949) and Yépez (2008), designing such a system imposes to determine the value of three engineering variables: the compressor horsepower H, the inside diameter of the pipe D and τ the pipe thickness. These variables must verify three engineering equations presented in Table 1 (first column). The compressor equation gives the power required to compress the gas flow from a given inlet pressure p_0 to a predefined outlet pressure $p_0 + \Delta p$ where Δp is the net pressure rise. The Weymouth equation models the pressure drop between the inlet pressure $p_0 + \Delta p$ measured after the compressor station, and the outlet one p_1, which is assumed to be equal to p_0. Lastly, concerns about the mechanical stability of the pipe impose a relation between the thickness τ and the inside diameter D.

Table 1. Engineering equations

Exact engineering equations	Approximate engineering equations
Compressor equation: [(a)] $$H = c_1 . \left[\left(\frac{p_0 + \Delta p}{p_0} \right)^b - 1 \right] Q$$	Approximate compressor equation: [(a)] $$H = c_1 b \frac{\Delta p}{p_0} Q$$
Weymouth flow equation: [(b)] $$Q = \frac{c_2}{\sqrt{l}} D^{8/3} \sqrt{(p_0 + \Delta p)^2 - p_1^2}$$	Approximate flow equation: [(b)] $$Q = \frac{c_2 p_0 \sqrt{2}}{\sqrt{l}} D^{8/3} \sqrt{\frac{\Delta p}{p_0}}$$
Mechanical stability equation: [(c)] $$\tau = c_3 D$$	Mechanical stability equation: [(c)] $$\tau = c_3 D$$

Notes: [(a) (b)] the positive constant parameters c_1, c_2 and b (with $b<1$) are detailed in Yépez (2008) for the USCS unit system. Elevation changes along the pipeline are neglected in the flow equation. [(c)] This equation follows the industry-standard practice and assumes that the pipe thickness equals a predetermined fraction c_3 of the inside diameter (e.g., c_3 =0.9% in Ruan et al. (2009 – p. 3044)).

We now combine these equations to construct an approximate production function. To our knowledge, the pressure rise Δp usually ranges between 1% and 30% of p_0, which leads to the first-

order approximations detailed in Table 1 (second column). Combining them, one can eliminate the relative pressure rise $\Delta p/p_0$ and obtain the following relation between the output Q and two engineering variables H and D:

$$Q = \sqrt[3]{\frac{2\left(c_2 p_0\right)^2}{c_1 bl}} D^{16/9} H^{1/3} . \tag{1}$$

This relation can be reformulated as a production function that gives the output as a function of two inputs: energy and capital. First, we let E denote the total amount of energy consumed by the infrastructure to power the compressor. By definition, the total amount of energy E is directly proportional to the horsepower H. Second, we let K denote the replacement value of the pipeline. We assume that the capital stock K is directly proportional to the pipeline total weight of steel S and let P_S denote the unit cost of steel per unit of weight. Hence, $K = P_S S$. The total weight of steel S required to build that pipeline is obtained by multiplying the volume of steel in an open cylinder by the weight of steel per unit of volume W_S:

$$S = l\pi \left[\left(\frac{D}{2} + \tau \right)^2 - \left(\frac{D}{2} \right)^2 \right] W_S , \tag{2}$$

where $\pi \approx 3.1416$ is the mathematical constant. Combining that equation with the mechanical stability equation in Table 1, the amount of capital expenditure related to the pipeline is as follows:

$$K = P_S l\pi D^2 \left[c_3 + c_3{}^2 \right] W_S . \tag{3}$$

This equation shows that the pipeline diameter is directly proportional to the square root of K, the amount of capital invested in the pipeline. So, the engineering equation (1) can readily be rewritten as a production function: $Q = B\, K^{8/9} E^{1/3}$, where B is a constant. To simplify, we rescale the output by dividing it by B and use this rescaled output thereafter. So, the Cobb-Douglas production function of a gas pipeline is:

$$Q^{\beta} = K^{\alpha} E^{1-\alpha} , \tag{4}$$

where the capital exponent parameter is $\alpha = 8/11$ and $\beta = 9/11$ is the inverse of the degree to which output is homogeneous in capital and energy.