

1 Economic Assessment of the Development of CO₂
2 Direct Reduction Technologies in Long-term
3 Climate Strategies of the Gulf Countries

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5 Electronic Supplemental Material

6 Frédéric Babonneau* Ahmed Badran[†] Maroua Benlahrech[‡]
7 Alain Haurie[§] Maxime Schenckery[¶] Marc Vielle^{||}

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9 **A Model formulation**

10 We report in this section the mathematical formulation of the meta-game
11 model used in this paper to design and assess burden sharing agreements.

12 **A.1 Model's equations**

13 **Variables and parameters**

14 $j \in \{1, \dots, m\}$: index of coalition;

15 $t \in \{1, \dots, T\}$: time periods;

16 $\delta(t)$: duration of time period t ;

*ORDECSYS, Switzerland and Escuela de Negocios, Universidad Adolfo Ibañez, Santiago, Chile. email:frederic.babonneau@uai.cl

[†]Qatar University, Doha, Qatar. email:a.badran@qu.edu.qa

[‡]Qatar University, Doha, Qatar. email:maroua.benlahrech@qu.edu.qa

[§]ORDECSYS and University of Geneva, Switzerland; GERAD, HEC Montréal, Canada. email:ahaurie@gmail.com

[¶]IFPEN, IFPSchool, Rueil-Malmaison, France. email:maxime.schenckery@ifpen.fr

^{||}École Polytechnique Fédérale de Lausanne, LEURE, Lausanne, Switzerland. email:marc.vielle@epfl.ch

- 17 B : global safety emission budget over the time horizon $[0, T]$;
- 18 θ_j : share of the global emission budget allocated to coalition j ;
- 19 $b_j = \theta_j B$: cumulative emission budget for coalition j at period 0;
- 20 $b_j(t)$ remaining emission budget for coalition j at end of period t ;
- 21 $\nu_j(t)$: K-T multiplier for global budget constraint of coalition j at period t ;
- 22 $\omega_j(t)$: supply of emission permits at period t by coalition j ;
- 23 $\Omega(t)$: total supply of emission permits at period t ;
- 24 $v_j(t)$: negative emission activity (CDR) by coalition j at period t ;
- 25 $v_j(0)$: negative emission activity (CDR) by coalition j at period 0;
- 26 $\kappa_j(v_j(t), t)$: cost of CDR for coalition j at period t ;
- 27 $q_j(t)$: abatement level by coalition j at period t ;
- 28 $\epsilon_j(t)$: BaU emission level by coalition j at period t ;
- 29 $e_j(t)$: emission level by coalition j at period t ;
- 30 $e_j(0)$: emission level by coalition j at period 0;
- 31 $\varpi_j(q_j(t), t)$: Abatement cost for coalition j at time t ;
- 32 $\mathbf{e}(t)$: vector of all m emission levels at period t ;
- 33 $\pi_j(\mathbf{e}(t), t)$: Net abatement cost (including changes in the terms of trade) for
34 coalition j at time t ;
- 35 $\gamma_j(\sum_{k=1}^m q_k(t), t)$: gains from the changes in terms of trade for coalition j at
36 time t ;
- 37 β_j : discount factor for coalition j equals 3%;
- 38 **Emissions from abatement.** This equation relates the abatement and
39 emission levels relative to BaU

$$e_j(t) = \epsilon_j(t) - q_j(t) \tag{1}$$

Emission budget constraints. Let $b_j(\tau)$ denote the remaining emission budget, for region j at the end of period τ , $\tau = 0, \dots, T-1$. We approximate the integral of net emissions up to period τ , using the trapezoidal method. The part of the emissions budget remaining at period τ is thus defined as

$$0 \leq b_j - \left(\frac{1}{2} \sum_{t=0}^{\tau-1} \delta(t+1)(\omega_j(t) + \omega_j(t+1) - v_j(t) - v_j(t+1)) \right),$$

$$j = 1, \dots, m, \quad \tau = 0, \dots, T-1. \quad (2)$$

40 By imposing non negative remaining budgets, we eliminate the possibility
41 for each “player” to perform short-selling of the future DAC activities.

This expression can also be rewritten

$$b_j - \left(\frac{1}{2} \delta(1)(\omega_j(0) - v_j(0)) + \frac{1}{2} \sum_{t=1}^{\tau-1} (\delta(t) + \delta(t+1))(\omega_j(t) - v_j(t)) \right. \\ \left. + \frac{1}{2} \delta(\tau)(\omega_j(\tau) - v_j(\tau)) \right) \geq 0, \quad j = 1, \dots, m, \quad \tau = 0, \dots, T-1. \quad (3)$$

42 Note that the modeling approach in [6, 5], is a Ramsey optimal growth
43 model with a constraint on SAT increase, expressed in terms of cumulative
44 emissions. In these cases, the optimal solution normally proposes some over-
45 shooting, with not insignificant negative emissions occurring at the end of
46 the planning horizon; and indeed, the higher the discount rate, the greater
47 the negative emissions at the end of the planning horizon. In contrast, our
48 approach uses an oligopoly game model, where each coalition strives to op-
49 timize the use of a given fixed emissions budget, over a planning horizon,
50 where a net-zero emissions regime should be reached at the end of the plan-
51 ning period. This will provide a natural end-of-period condition: net-zero
52 emissions for the whole world. However, at each intermediate period, a neg-
53 ative remaining budget for one coalition would allow it to supply emission
54 rights on the current market as CO₂ they promise to capture in the future:
55 in other words, short selling, associated with high risk and temptation for
56 each player to cheat. Under these conditions, and to ensure that each permit
57 supplied corresponds to existing abatements or CO₂ capture, then, it would
58 be necessary to forbid short selling.

59 **Net-zero emissions in the final period.** At the end of the planning
60 horizon one must reach a net-zero emission regime. So there should be a

61 coupled constraint of the form.

$$\sum_j (v_j(T) - e_j(T)) \geq 0. \quad (4)$$

62 However, this constraint will probably be redundant with the emission bud-
63 get constraints and we will not consider it.

64 **Emissions trading.** An international carbon market determines a price
65 and emissions levels.

$$p(t) = \frac{\partial}{\partial q_j(\cdot)} \varpi_j(q_j(t), t) = -\frac{\partial}{\partial e_j(\cdot)} \varpi_j(\epsilon_j(t) - e_j(t), t) \quad (5)$$

$$\Omega(t) = \sum_{k=1}^m e_k(t); \quad j = 1, \dots, m. \quad (6)$$

66 The price and emission levels are thus functions of the total permit supply
67 $\Omega(t)$, thus denoted $\tilde{\mathbf{e}}(\Omega(t), t)$ and $\tilde{p}(\Omega(t), t)$, respectively.

68 As shown in Helm [4], the derivatives w.r.t. Ω of price and emission
69 levels are given by

$$\tilde{p}'(\Omega, t) = \frac{1}{\sum_{j=1}^m \frac{1}{\frac{\partial^2 \varpi_j(q_j, t)}{\partial q_j^2}}} \quad (7)$$

$$\tilde{e}'_j(\Omega, t) = \frac{1}{\sum_{k=1}^m \frac{\frac{\partial^2 \varpi_j(q_j, t)}{\partial q_j^2}}{\frac{\partial^2 \varpi_j(q_k, t)}{\partial q_k^2}}} \quad (8)$$

70 respectively. Since $\Omega(t) = \sum_{j=1}^m \omega_j(t)$ the derivatives w.r.t. $\omega_j(t)$ are the
71 same as the derivatives w.r.t. $\Omega(t)$.

72 **Payoffs.** The periodic net cost to coalition j includes the abatement cost
73 plus the cost of buying permits on the market (negative if selling) and is
74 given by

$$\psi_j(t) = [\pi_j(\tilde{\mathbf{e}}(\Omega(t), t) + \kappa_j(v_j(t), t) - \tilde{p}(\Omega(t), t)(\omega_j(t) - e_j(\Omega(t), t))), \quad (9)$$

75 where

$$\pi_j(\mathbf{e}(t), t) = \varpi_j(q_j(t), t) - \gamma_j\left(\sum_k p_k(t), t\right). \quad (10)$$

The payoff coalition j is defined by the integral of the discounted periodic costs

$$J_j(\cdot) = \frac{1}{2}\delta(1)\psi_j(0) + \frac{1}{2}\sum_{t=1}^{T-1}\beta_j^t(\delta(t) + \delta(t+1))\psi_j(t) + \frac{1}{2}\beta_j^T\delta(T)\psi_j(T),$$

$$j = 1, \dots, m. \quad (11)$$

76 We assume that the supply of permits and the DAC activities of each coalitions are strategically defined as the open-loop Nash equilibrium for the
77 game defined by payoffs (11) and constraints (1)-(8).
78

79 A.2 Nash equilibrium conditions

We write now the first order conditions for a Nash equilibrium solution. The existence of a solution is implied by the convexity of the cost functions. Denoting $\nu_j(t)$ the K-T multiplier of the emission budget constraint (3) for coalition j , we may write the Lagrangian for each player j as given by

$$\mathcal{L}_j(\cdot) = \frac{1}{2}(\delta(1)\psi_j(0) + \beta_j^T\delta(T)(\psi_j(T))) + \frac{1}{2}\sum_{t=0}^{T-1}\beta_j^t(\delta(t) + \delta(t+1))(\psi_j(t) +$$

$$\nu_j(t)(b_j - \frac{1}{2}\sum_{s=0}^{t-1}\delta(s+1)(\omega_j(s) + \omega_j(s+1) - v_j(s) - v_j(s+1)))$$

$$j = 1, \dots, m. \quad (12)$$

Complementarity conditions for $\omega_j(t)$

$$0 \leq \beta_j^t \frac{\partial}{\partial \omega_j(t)} [\pi_j(\tilde{\mathbf{e}}(\Omega(t), t) - \tilde{p}(\Omega(t), t)(\omega_j(t) - e_j(\Omega(t), t)))] + \nu_j \quad (13)$$

$$0 \leq \omega_j(t) \quad (14)$$

$$0 = \omega_j(t) \left\{ \beta_j^t \frac{\partial}{\partial \omega_j(t)} [\pi_j(\tilde{\mathbf{e}}(\Omega(t), t) - \tilde{p}(\Omega(t), t)(\omega_j(t) - e_j(\Omega(t), t)))] + \nu_j \right\}. \quad t = 1 \dots T \quad (15)$$

Developing the expression

$$\begin{aligned}
\frac{\partial}{\partial \omega_j(t)} [\pi_j(\tilde{\mathbf{e}}(\Omega(t), t) - \tilde{p}(\Omega(t), t)(\omega_j(t) - e_j(\Omega(t), t)))] = \\
\frac{\partial}{\partial \sum_k q_k(t)} \gamma_j(\sum_k q_k(t), t) \frac{\partial}{\partial \omega_j(t)} (\sum_{k=1}^m e_k(\Omega(t), t)) \\
- (\frac{\partial}{\partial q_j(t)} \varpi(q_j(t), t) - \tilde{p}(\Omega(t), t)) \frac{\partial}{\partial \omega_j(t)} e_j(\Omega(t), t) \\
- \tilde{p}(\Omega(t), t) - \frac{\partial}{\partial \omega_j(t)} \tilde{p}(\Omega(t), t)(\omega_j(t) - e_j(\Omega(t), t)), \quad (16)
\end{aligned}$$

and using the relations $\frac{\partial}{\partial q_j(t)} \varpi(q_j(t), t) = \tilde{p}(\Omega(t), t)$ and $\sum_{k=1}^m e_k(\Omega(t), t) = \Omega(t)$ that hold on the emission permit market the complementarity condition (15) can be rewritten more simply

$$\begin{aligned}
\omega_j(t) \left\{ -\beta_j^t \left[-\frac{\partial}{\partial \sum_k q_k(t)} \gamma_j(\sum_k q_k(t), t) + \tilde{p}(\Omega(t), t) \right. \right. \\
\left. \left. + \frac{\partial}{\partial \omega_j(t)} \tilde{p}(\Omega(t), t)(\omega_j(t) - e_j(\Omega(t), t)) \right] + \nu_j \right\} = 0. \quad (17)
\end{aligned}$$

Complementarity conditions for $v_j(t)$

$$0 \leq \beta_j^t \frac{\partial}{\partial v_j(t)} \kappa_j(v_j(t), t) - \nu_j \quad (18)$$

$$0 \leq v_j(t) \quad (19)$$

$$0 = v_j(t) \left\{ \beta_j^t \frac{\partial}{\partial v_j(t)} \kappa_j(v_j(t), t) - \nu_j \right\}. \quad (20)$$

Complementarity conditions for $\nu_j(t)$

$$0 \leq b_j - \frac{1}{2} \sum_{s=0}^{t-1} \delta(s+1)(\omega_j(s) + \omega_j(s+1) - v_j(s) - v_j(s+1)) \quad (21)$$

$$0 \leq \nu_j(t) \quad (22)$$

$$\begin{aligned}
0 = \nu_j(t) \left\{ b_j - \frac{1}{2} \sum_{s=0}^{t-1} \delta(s+1)(\omega_j(s) + \omega_j(s+1) - v_j(s) - v_j(s+1)) \right\} \\
, j = 1, \dots, m. \quad (23)
\end{aligned}$$

80 A.3 Model calibration - CO₂ emissions and payoff functions

81 We use the GEMINI-E3 model [2, 3] to calibrate the dynamic game model.
 82 GEMINI-E3 is a worldwide multi-country, multi-sector, computable gen-
 83 eral equilibrium (CGE) model that has been specifically designed to assess
 84 energy and climate change policies. GEMINI-E3 is used to compute the
 85 CO₂ emissions and economic variables within the business as usual (BaU)
 86 scenario and calibrate the payoff functions (π_j). The methodology used
 87 to calibrate our game theory model using an applied CGE is detailed in
 88 our previous papers, e.g. see Appendix 2 in [1]. In short, various climate
 89 policies are simulated by GEMINI-E3, then we perform econometric esti-
 90 mations of the abatement cost ($\varpi_j(q_j(t), t)$) and gains from term of trade
 91 ($\gamma_j(\sum_{k=1}^m q_k(t), t)$) functions. However, the time horizon of GEMINI-E3 is
 92 limited to the first part of our century (i.e. up to 2050), therefore we have to
 93 implement a procedure extending the variables for the years 2070 and 2100.
 94 We use a versatile representation based on a steady state growth approach
 95 for the end of our century.

$$\begin{aligned}
 \frac{gdp_j(t) - gdp_j(t-1)}{gdp_j(t-1)} &= \frac{pop_j(t) - pop_j(t-1)}{pop_j(t-1)} \cdot (1 + \nu_j^1(t))^{\delta(t)} \\
 \frac{e_j(t) - e_j(t-1)}{e_j(t-1)} &= \frac{gdp_j(t) - gdp_j(t-1)}{gdp_j(t-1)} \cdot (1 + \nu_j^2(t))^{\delta(t)} \\
 \nu_j^1(t) &= \nu_j^1(t-1) - \delta(t) \cdot (\nu_j^1(t-1) - \nu_j^1(T)) / (\delta(T-1) + \delta(T)) \\
 \nu_j^2(t) &= \nu_j^2(t-1) - \delta(t) \cdot (\nu_j^2(t-1) - \nu_j^2(T)) / (\delta(T-1) + \delta(T)) \\
 \nu_j^1(T) &= \nu^1 \quad \forall j \\
 \nu_j^2(T) &= \nu^2 \quad \forall j
 \end{aligned} \tag{24}$$

96 First, we select a demographic scenario among the projections done by
 97 the United Nations [7] and determine the working population¹ ($pop_j(t)$).
 98 Then, we follow a production function approach linking GDP per capita
 99 ($gdp_j(t)/pop_j(t)$) to a total productivity factor (TFP) $\nu_j^1(t)$. We assume
 100 that for each region the TFP converges to a common value (ν^1) at the end
 101 of our century. Finally, we assume that for each region CO₂ emissions per
 102 GDP ($e_j(t)/gdp_j(t)$) decrease with an annual rate that converges to a single
 103 value ν^2 . Thus we can simulate various BaU scenarios by setting a value for
 104 the three parameters defined above, demographic scenario, ν^1 and ν^2 .

105 The abatement functions ($\varpi_j(q_j(t), t)$) are extrapolated for the years
 106 2070 and 2100 by assuming a proportionally rule with respect to the level

¹Male and female population aged from 20 to 64.

107 of abatement for the year 2050. The GTT functions $(\gamma_j(\sum_{k=1}^m q_k(t), t))$ in
108 2070 and 2100 are supposed unchanged with respect to 2050 figures.

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