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LiDAR and SCADA data processing for interacting wind turbine wakes with comparison to analytical wake models

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Abstract: This study is a follow up on a previous one carried out within the frame of the French project SMARTEOLE, during which, a ground-based scanning LiDAR measurement campaign was conducted in the onshore wind farm of Sole du Moulin Vieux. That previous study focused on the wakes of two wind turbines that experienced different degrees of interaction depending on the incoming wind direction, through the processing of LiDAR measurements. The measurement duration (7 months) ensured the statistical convergence of the ensemble-averaged flow fields obtained after holding a categorisation process based on the wind speed at hub height, wind direction, and atmospheric stability, where only near-neutral stability conditions were considered. The present study focuses on integrating the operational data of the wind turbines through SCADA processing to complement the LiDAR wake field observations and to be used as an input for analytical wake models. First, the correlation between the atmospheric stability, deduced from MERRA-2 dataset, and the free-stream turbulence intensity, measured by the wind turbines' anemometers, is studied for different wind speed ranges. It is observed that the turbulence intensity tends towards a consistent value as the atmospheric stability approaches near-neutral stability conditions, giving confidence into the applied strategy of data categorisation based on MERRA-2 outputs. The influence of the degree of wake interaction on the wake added turbulence, the velocity and power deficits between both turbines is assessed. Clear trends between the wake added turbulence and both the velocity and power deficits are detected. Consequently, two fitting laws are proposed. Then, different analytical wake models and wake superposition methods are fed with the operational data deduced from the processed SCADA data, and are used for predicting the evolution of the velocity deficit within the wake. Some statistical metrics are used for error quantification of the different engineering wake models compared to the scanning LiDAR measurements, used as reference, and Blondel and Cathelain produces the closest results to the field measurements.

Keywords: Wind energy; Atmospheric boundary layer; Wind turbine wake; Wake interactions; Wake models; Wake superposition

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1. Introduction

The urgent need for designing large wind farm arrays requires a deep understanding of the wind turbine (WT) wakes, as their quantification and modelling are challenging, not only for the scientific community, but also for wind farm developers. When a WT operates within the wake of another, a reduction in its incoming wind speed occurs, which causes power losses that can reach 20% of the power generated from the undisturbed ones [1–3]. Furthermore, WT's experience fatigue loads due to the increased turbulence intensity within the wakes, leading to the reduction of their lifetime. Therefore, any improvements in the understanding and characterisation of WT wakes would reduce the uncertainties in the power losses [4], lead to an increase the power production, hence, increase the economic viability of wind energy [5].

When air flows over a WT, it is affected by its presence both upstream and downstream [5,6]. Consequently, the area surrounding a WT is divided into an upstream region called *induction zone* reaching up to a distance of $2D_r$, D_r being the rotor diameter, whereas the downstream one is called *wake region*. In the induction zone, wind speed is progressively reduced upstream of the turbine [5,7,8]. The wake is composed of two sub-regions; the near-wake region that lies directly downstream the turbine with a length of $2 - 4D_r$ [5], and the far-wake region that starts after the near-wake region. The location and extent of the transition between near- and far-wake regions are not fully identified and depend on the environmental conditions, WT operational points, etc.

There are several approaches to study the flow over WT's. They include analytical models [5, 9–15], computational fluid dynamics (CFD) [16–20], wind tunnel experiments [21–23] and field experiments [24–26]. Analytical modelling presents a simple approach that requires low computational cost and is thus preferred in industry for wind farm design. CFD on the other hand, is a more sophisticated and expensive approach, yet gives more physical insight. Wind tunnel experiments provide valuable information on the flow structure of WT wakes in homogeneous and boundary layer flows, and very useful datasets for the validation of analytical and CFD models. However, it is always difficult to resemble the real process in the lab due to the applied trade-offs (e.g., full similarity issues). Field measurements provide the real-life insights, but they often lack statistical convergence because of the continuous changes in the atmospheric conditions and terrain properties, which affects the wake characteristics significantly [1]. Therefore, long term field observations are essential for proper wake characterisation [1,27,28]. It is also difficult to have high spatial precision measurements.

Field measurements used to be collected using anemometers mounted either on meteorological masts or on the turbines themselves to measure WT wakes. Nowadays, scanning wind Light Detection And Ranging (LiDAR) is used more often [1,5], which helps overcome the drawbacks of using anemometers; including poor space resolution and limited wind direction [1].

Regarding analytical wake modelling, there are several models that are simple and computationally inexpensive, yet superior in capturing the physics when compared to empirical models (i.e., obtained by data fitting) such as Zhang *et al.* [29], Iungo and Porté-Agel [30], and Aitken *et al.* [31]. This is due to the fact that these analytical wake models such as Jensen [9], Ainslie [10], Larsen [11], Frandsen *et al.* [12], Bastankhah and Porté-Agel [13], Qian and Ishihara [14], and Blondel and Cathelain [15] are derived from flow governing equations; either mass conservation equation only or mass and momentum conservation equations together [5]. The models that attracted attention the most (Jensen [9], Frandsen *et al.* [12], Bastankhah and Porté-Agel [13]), and a new promising one (Blondel and Cathelain [15]), are used in this study.

Jensen [9] assumed a top-hat shape for the wake velocity deficit profile which was based on solving the mass conservation equation only [5,9,13,15,32]. Duc *et al.* [33] proposed a tuning procedure to improve the accuracy of Jensen wake model based on locally updating the wake decay constant at each WT depending on the turbulence intensity measured by the nacelle anemometer. Following that procedure, the modified Jensen wake model describes more precisely the individual wake deficit at each WT as reported in [33].

The same assumption of the top-hat wake profile was pursued by Frandsen *et al.* [12], however, the model is based on satisfying both the mass and momentum conservation equations [5,12,13,15,32]. The assumption of the top-hat profile is very bold and unrealistic, as studies showed that the streamwise velocity deficit profiles tends towards an axisymmetric Gaussian distribution in the far-wake region [34]. As a result, Bastankhah and Porté-Agel [13] recently developed an extension of the Jensen model, replacing the overly simplified top-hat profile with a self-similar Gaussian profile, which presents a closer approximation for classic far-wake profile [5,15,32].

Although Jensen [9] and Frandsen *et al.* [12] models are meant to describe the wake profile in the far-wake, yet there is no explicit limitation on applying them for the whole wake, while Bastankhah and Porté-Agel model [13] considered the far-wake region only, none of these models accounted for the near-wake region. Therefore, the choice of either the top-hat shape only or the Gaussian shape only is not quite accurate [15], as the velocity deficit increases till reaching a maximum value, then decreases, due to the turbulent mixing. It was also observed that the velocity profile evolves downstream of the wind turbine from a top-hat shape to a Gaussian shape in the far wake [15,21,35,36]. There was no wake model providing a full demonstration of the whole wake region till the recent contributions from Qian and Ishihara [14]. They proposed a modified version of the Bastankhah and Porté-Agel model [13] that improves the velocity deficit prediction in the near wake. In this updated model, a corrective term is added to predict realistic values for near-wake wind speeds. However, this corrective term causes a violation of the mass and momentum conservation [15].

Blondel and Cathelain [15] developed a model to overcome the violation of the conservation equations by replacing the Gaussian shape with a super-Gaussian shape, which tends towards a top-hat shape for high values of the super-Gaussian order n (near-wake conditions), and towards the traditional Gaussian shape for $n = 2$ (far-wake conditions). As a result, it was reported that the wake velocity profiles were more consistent with observations, the velocity deficit had the expected form, and mass and momentum conservation equations were preserved [15]. However, the model required further calibration and validation, and therefore, a newer and more accurate version of the model was later proposed in Cathelain *et al.* [37], which is used in this paper, (see Appendix A for the comparison between both versions).

The aforementioned wake models address the wake of a single WT. However, to describe the wake within a wind farm where turbines operate in the wake of each other, these wake models should be coupled with superposition methods to account for the interaction among multiple wakes [5]. Different superposition methods have been proposed, and new ones are being developed actively. Lissaman [36] method is based on the principle of linear superposition of velocity deficit defined with respect to the incoming boundary layer flow speed. Afterwards, Katić *et al.* [38] followed referring the velocity deficit to the incoming boundary layer flow speed similar to Lissaman [36], however, with a different superposition principle that involved the linear superposition of energy deficit. This superposition principle was also adopted later by Voutsinas *et al.* [39], yet the velocity deficit was defined with respect to the incoming flow velocity for the turbine under study. Subsequently, Niayifar and Porté-Agel [40] proposed a method with a superposition principle similar to Lissaman's [36], and a velocity deficit reference similar to Voutsinas *et al.* [39]. Later, Zong and Porté-Agel [41] developed an iterative method relying on mass and momentum conservation considerations. Recently, Bastankhah *et al.* [42] developed a new analytical model that provides an explicit wind farm solution also based on mass and momentum conservation considerations.

The present study is considered as a continuation to the work done in Torres-Garcia *et al.* [1]. They processed experimental field wake measurements obtained during a 7-month ground-based scanning LiDAR measurement campaign, with a set-up that enables to capture two WT wakes up to $10D_r$ downstream and for a wind sector of 40° . Depending on the wind direction, WT wakes experienced different interaction levels ranging from full interaction till zero interaction. This

long measurement campaign ensured the satisfaction of statistical convergence, which led to obtaining exploitable ensemble-averaged results on wake properties. After the processing of measurements, different wake characteristics were investigated such as wake centre-line position, velocity deficit and the wake meandering associated with large-scale turbulent structures in the incoming atmospheric flow. The associated effective operational WT conditions were not considered. Supervisory Control and Data Acquisition (SCADA) data processing is therefore performed following the procedures proposed in [1]. The processed SCADA data is first analysed, then used as an input for the analytical wake models with the objective of checking their validity and accuracy when compared to field measurements, and at the same time holding a clear comparison between these models.

In the following, section 2 provides a description of the equipment and experimental set-up. In section 3, the measurement processing and classification methods are described. In section 4, the main results, and observations of both studies; data analysis and wake modelling, are discussed. Finally, in section 5, a summary and recommendations for future research are provided.

2. Experimental set-up

The field measurement campaign took place in the Sole du Moulin Vieux (SMV) onshore wind farm, owned by Engie Green, located in Ablaincourt-Pressoir municipality in the north of France in Picardy region. The wind farm consists of seven WTs, as shown in Fig. 1, sited from north to south with approximate spacing of around $3.5D_r$ apart. The WTs are SENVION MM82, with a diameter of $D_r = 82 \text{ m}$, a hub height of $z_h = 80 \text{ m}$; and a nominal power of 2050 kW for a nominal wind speed of 14.5 m.s^{-1} , in addition to a cut-in wind speed of 3.5 m.s^{-1} . At the hub height z_h of WTs SMV6 and SMV5, a wind vane and an anemometer registered the conditions at WT locations using SCADA system [1,25].

A pulsed scanning LiDAR (Windcube 200S) developed by Leosphere, with a speed accuracy of 0.1 m.s^{-1} , was positioned 1320 m east from WT-SMV4 at ground level. The LiDAR was programmed to hold scans covering azimuth angles ranging between 243° and 273° (meteorological coordinates), as shown in Fig. 1.



Figure 1: The wind farm composed of seven WTs marked with red dots. A meteorological mast (A) is located in the surrounding area. The location of the LiDAR is marked with B. The shadowed area, C, is the top view of the area scanned by the LiDAR [1].

3. Methods

As previously mentioned, the data handling procedure of Torres-Garcia *et al.* [1] is followed in this study. They considered LiDAR and the Modern-Era Retrospective analysis for Research and Applications version 2 (MERRA-2) datasets. SCADA data is added to the group to have a broader description of the overall situation and be able to hold comparisons. Full details about the LiDAR data processing can be found in [1].

Before categorising the SCADA data, an hourly-averaging is performed to match the hourly MERRA-2 dataset, since SCADA data is provided as 10-minute average by default. Afterwards, the categorisation process is held according to specific environmental wind flow conditions obtained from MERRA-2. These conditions include: 1) Atmospheric stability (near-neutral), 2) Environmental wind speed ($13 \pm 1.95 \text{ m.s}^{-1}$ at 50 m height), and 3) Wind direction (four directions; 207° , 220° , 233° and $246^\circ \pm 6.4^\circ$). An additional filter based on grid availability ensures that only fully operating WT's are post-processed. Only 0.2% of the data was rejected due to this filter.

The ABL stability is described by Monin-Obukhov Length (L) obtained from MERRA-2. The near-neutral stability range considered for this study is $|L| \geq 300 \text{ m}$ and is based on the atmospheric stability classifications defined in Wharton *et al.* [43].

The wind speed (WS) used for the categorisation process is the one obtained from MERRA-2 at 50 m above the ground. A reference WS $u = 13 \text{ m.s}^{-1}$ was considered. In such a configuration, stronger wake effects will occur because the WT will be working close to its nominal WS [1].

Four WDs were used in the study to consider different levels of interaction; starting from full interaction in case of 207° , where a wake is generated downstream of WT-SMV6 and goes all the way through WT-SMV5 whose wake includes the superimposed effects of both WT-SMV6 and WT-SMV5. As for the other three WDs, the level of interaction decreases gradually till reaching the case of zero interaction when wind flows from a direction of 246° .

3.1. Velocity deficit

There are many suggestions on how to calculate the velocity deficit (VD). Here, VD is defined as in [1]:

$$VD_{LiDAR}(x_{WD}, z_{WD}) = \frac{u_{hub} - u_{min}(x_{WD}, z_{WD})}{u_{hub}} \quad (1)$$

where u_{hub} is the reference velocity at hub height extracted from the undisturbed environmental vertical velocity profile, while u_{min} is the minimum velocity at a distance (x_{WD}) downstream of the wind turbine and at an altitude z_{WD} where the LiDAR measurements were obtained. To compare the LiDAR VD evolution with the VD obtained from the engineering wake models, this formula is adapted as follows:

$$VD_{Model}(x_{WD}, z_{WD}) = \frac{u_{SMV6} - u_{min}(x_{WD}, z_{WD})}{u_{SMV6}} \quad (2)$$

where u_{SMV6} is the WS measured by the anemometer on WT-SMV6.

Another definition of VD is also employed, in the data processing part of the results. It is different to Eq. (1) because it uses the spatially resolved LiDAR classified data, thus, evolves with the downstream distance (x_{WD}). Here, VD_{SCADA} is calculated from the categorised SCADA data at the fixed locations of both WT's SMV6 and SMV5:

$$VD_{SCADA} = \frac{u_\infty - u_w}{u_\infty} = \frac{u_{SMV6} - u_{SMV5}}{u_{SMV6}} \quad (3)$$

where u_∞ is the undisturbed upstream velocity, while u_w is the downstream velocity in the wake of the undisturbed WT. u_∞ was taken equal to u_{SMV6} , and u_w was taken equal to u_{SMV5} , which is the WS measured by the anemometer on WT-SMV5. VD_{SCADA} depends on the position vector, $X = (x_{WD}, y_{WD}, z_{WD})$, although it is not explicitly shown in Eq. (3).

3.2. Power deficit

The power deficit (PD) between the two WT's SMV5 and SMV6 is calculated similarly:

$$PD_{SCADA} = \frac{P_{SMV6} - P_{SMV5}}{P_{SMV6}} \quad (4)$$

where P_{SMV6} is the power generated by WT-SMV6, while is the power generated by WT-SMV5.

3.3. Turbulence intensity and wake added turbulence

The turbulence intensity (I_u) defines the level of velocity fluctuations due to turbulence within the flow. It is the ratio between the standard deviation of the streamwise WS (σ_u), which refers to the fluctuations around the mean value, and the mean value of streamwise WS (u) as follows for both WT's SMV6 and SMV5:

$$\begin{aligned} I_{u_{SMV6}} &= \sigma_{u_{SMV6}} / u_{SMV6} \\ I_{u_{SMV5}} &= \sigma_{u_{SMV5}} / u_{SMV5} \end{aligned} \quad (5)$$

Meanwhile, the wake added turbulence intensity is the additional turbulence induced by the WT wake, and is obtained from [5] as follows:

$$\Delta I = \sqrt{I_{wake}^2 - I_{\infty}^2} = \sqrt{I_{u_{SMV5}}^2 - I_{u_{SMV6}}^2} \quad (6)$$

where I_{wake} is the streamwise turbulence intensity in the wake measured by WT-SMV5, while $I_{u_{SMV6}}$ is the streamwise turbulence intensity measured by WT-SMV6 and considered as the free-stream turbulence intensity (I_{∞}).

3.4. Wake modelling

A mix of empirical and analytical models was used for the study. It included Aitken *et al.* [25], Jensen [9], Frandsen *et al.* [12], Bastankhah and Porté-Agel [13] because they were the ones that attracted the most attention according to [4]. That is in addition to the newly proposed model from Blondel and Cathelain [15, 37], (see Appendix A).

To analytically model wind farm flows, the aforementioned wake models are coupled with wake superposition methods to account for the wake interaction. Different techniques are available to estimate the velocity, $u(X)$, at a given position $X = (x, y, z)$ in a wind farm. The ones employed in this paper are Lissaman [36], Katić *et al.* [38], Voutsinas *et al.* [39], and Niayifar and Porté-Agel [40], (see Appendix B). The recent method from Zong and Porté-Agel [41] was not used in the current study because it is not represented in an explicit form and follows an iterative process to be incorporated, which is not computationally friendly [42]. The novel method from Bastankhah *et al.* [42] was not employed since it was mainly developed for the Gaussian model, and therefore, it would need to be adapted to the other wake models, which is not straightforward and is still under investigation.

3.5. Performance measures

A group of statistical metrics are used to measure a model performance by comparing model predictions and measurements, then quantifying the errors [44]. The ones included in the study are Fractional Bias (FB), Geometric Mean Bias (MG), Normalised Mean Squared Error (NMSE), Geometric Variance (VG), Correlation Coefficient (R), and the fraction of predictions within a factor of two of observations (FAC2), see Appendix C. A perfect model would have MG, VG, R, and FAC2 = 1; and FB and NMSE = 0. However, there is no such thing as a perfect model, because of the influence of random processes involved in the system [44,45].

4. Results

4.1. Data processing

Table 1 illustrates the ensemble-average of the WD values obtained from LiDAR measurements depicted in [1], in addition to MERRA-2, and post-processed SCADA datasets in the four WD cases. At the same time, it ensures the validity of the categorisation process held by comparing the average WD values from both SCADA and LiDAR. There is only a slight

difference between the SCADA values shown in this table and their corresponding LiDAR values, which confirms the validity of the process. This insignificant difference might result from the application of the grid availability condition during categorisation of the SCADA dataset. However, there is a very good agreement between the results that are within a range of about $\pm 2^\circ$, similarly as it was mentioned in [1]. The statistical convergence of SCADA data was ensured in a similar way that was reported in [1] for LiDAR data.

Table 1: Summary of characteristics of the four cases after categorisation. The cases were extracted from periods when the atmosphere was neutral according to the MERRA-2 dataset.

Case	Interaction level	Wind Direction (WD) [$^\circ$]			
		LiDAR	MERRA-2	SCADA	
				SMV6	SMV5
207 $^\circ$	Full	208 $^\circ$	209 $^\circ$	210.3 $^\circ$	210.2 $^\circ$
220 $^\circ$	Partial	219 $^\circ$	220 $^\circ$	217.8 $^\circ$	218 $^\circ$
233 $^\circ$	Weak	233 $^\circ$	235 $^\circ$	231 $^\circ$	229.9 $^\circ$
246 $^\circ$	Zero	246 $^\circ$	245 $^\circ$	243.4 $^\circ$	241.7 $^\circ$

Table 2 shows the ensemble-averaged values for different variables after categorisation. Some of these variables such as the average streamwise WS and I_u , are fed into the analytical wake models as inputs to obtain the VD in the streamwise direction within the wake. Some variables could be obtained from both MERRA-2 and SCADA datasets, while others were only obtained from either of them. There is an expected difference between the average WS values obtained from MERRA-2 and SCADA for WT-SMV6. That is because both datasets have different sources of uncertainty, and because the altitude at which the WS is given is different (50 m for MERRA-2 and 80 m for SCADA). There is a significant difference between the values obtained from WT-SMV6 and WT-SMV5, which demonstrates the wake interaction effect. Also, in the case of WS and power, there is a significant decrease between values for the two WTs when wakes fully interact ($WD = 207^\circ$), and this decrease reduces as the degree of interaction decreases. Regarding I_u , there is an increase in case of full interaction, which fades away as the interaction level decreases. The average value of L in the four WD cases shows that they lie in near-neutral ABL.

Table 2: Ensemble-averaged values of variables of interest after categorisation.

Wind Speed (WS) [m.s $^{-1}$]				Power [kW]		I_u		L [m]
Case	MERRA-2	SCADA		SCADA		SCADA		MERRA-2
		SMV6	SMV5	SMV6	SMV5	SMV6	SMV5	
207 $^\circ$	12.88	11.79	9.63	1683.55	1202.72	0.129	0.183	887.43
220 $^\circ$	12.74	11.80	10.12	1709.82	1343.66	0.125	0.163	1195.46
233 $^\circ$	12.48	11.84	11.41	1787.20	1721.89	0.107	0.114	1774.96
246 $^\circ$	12.61	11.45	11.16	1721.72	1723.61	0.103	0.104	1025.11

The average streamwise WS values shown in Table 2 are used to calculate the thrust coefficient (C_T) of the WTs, which is an important parameter in the analytical wake models. C_T is computed using linear interpolation with the help of the C_T table provided in the WT manufacturer's data sheet. Table 3 shows the calculated C_T values to be fed into the wake models together with the values of WS and I_u from Table 2.

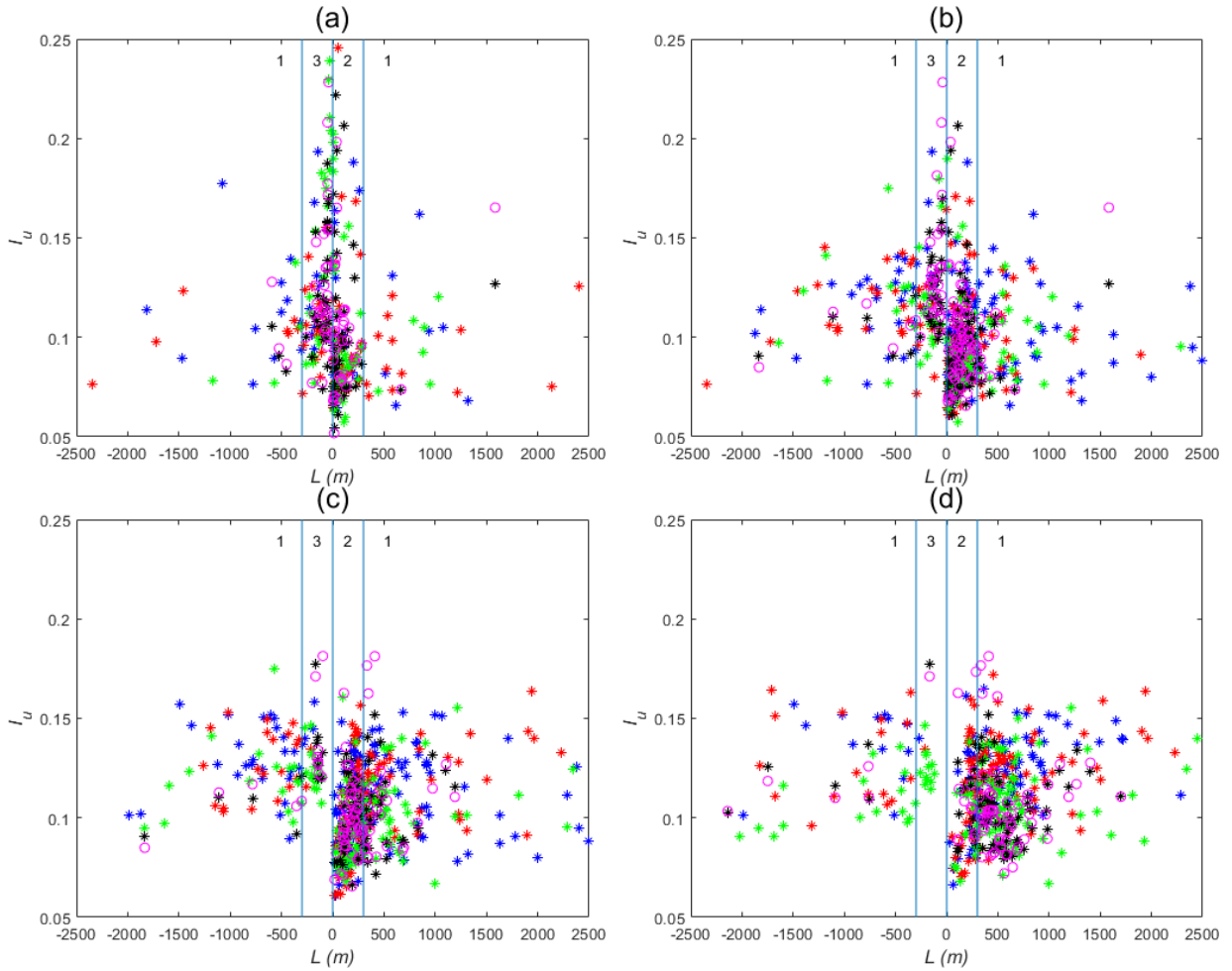
Table 3: Thrust coefficient (C_T) of both wind turbines in each wind direction case.

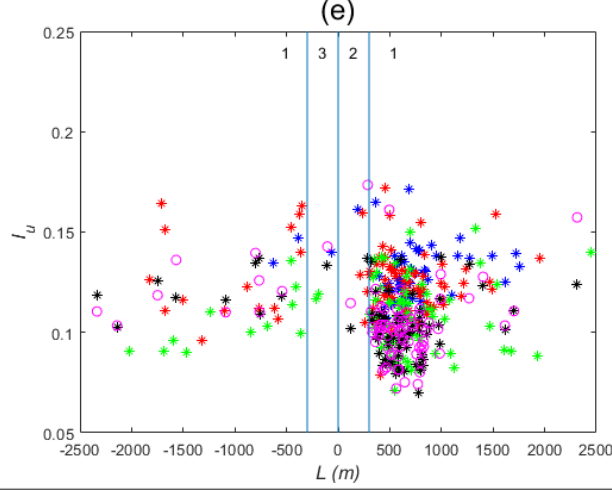
C_T	Case			
	207 $^\circ$	220 $^\circ$	233 $^\circ$	246 $^\circ$

SMV6	0.5345	0.5342	0.5291	0.5765
SMV5	0.7419	0.7094	0.5811	0.6111

4.2. Relationship between atmospheric stability and turbulence intensity

To study the relationship between the atmospheric stability, described by L , and the free-stream turbulence intensity (I_u), three atmospheric stability cases are included with their corresponding L ranges: 1) Near-neutral ($|L| \geq 300 \text{ m}$), 2) Stable ($0 \leq L < 300 \text{ m}$), 3) Convective ($-300 \text{ m} < L < 0$). Configurations with wake interaction are neglected in this specific study, thus, the 4 WDs for WT-SMV6, and $WD = 246^\circ$ for WT-SMV5 are only considered. Five WSs are considered; 5 m.s^{-1} , 7 m.s^{-1} , 9 m.s^{-1} , 11 m.s^{-1} and 13 m.s^{-1} , with a tolerance of ± 1.95 . By including the different WSs, different degrees of flow advection are considered. Generally, in strong winds, atmospheric stability conditions tend to near-neutral, whereas stable and unstable winds occur more in case of lighter winds.





* WD207-SMV6 * WD220-SMV6 * WD233-SMV6 * WD246-SMV6 ○ WD246-SMV5

Figure 4: Relationship between atmospheric stability (L) and free-stream turbulence intensity (I_u) at different wind speeds for zero-wake interaction cases, where each stability range is bounded by a light blue vertical line, and the numbers 1, 2 and 3 correspond to the near-neutral, stable and convective stability ranges respectively. (a) $WS = 5 \text{ m.s}^{-1}$, (b) $WS = 7 \text{ m.s}^{-1}$, (c) $WS = 9 \text{ m.s}^{-1}$, (d) $WS = 11 \text{ m.s}^{-1}$, (e) $WS = 13 \text{ m.s}^{-1}$

Fig. 4 illustrates the relationship between L and I_u measured by the WT's, at different WS ranges for zero-wake interaction cases. It is obvious in the figure that, as expected, the data points, migrate from near-neutral atmospheric stability at strong winds (higher WSs), towards stable and unstable stability configurations as the WS decreases. Furthermore, a trend between I_u and L , can be detected when the WS changes from 5 m.s^{-1} to 13 m.s^{-1} . I_u tends towards a consistent value and presents less scatter as the atmospheric stability goes towards the neutral stability configurations. This analysis gives confidence on the strategy, which was applied in the previous [1] and the present paper, of filtering the database according to the Monin-Obukhov Length (L) deduced from MERRA-2 to extract near-neutral configurations, since the actual turbulence intensity measured onsite for the selected periods presents a homogenous and coherent level.

4.3. Wake added turbulence intensity

The effect of the wake added turbulence intensity (ΔI) on the velocity deficit (VD) and the power deficit (PD) was investigated. The VD and PD are obtained from the categorised SCADA data in near-neutral atmospheric stability conditions.

4.3.1. Wake added turbulence intensity and velocity deficit

The relationship between the VD, obtained from Eq. (3), and ΔI , calculated using Eq. (6), was investigated (Fig. 5). Fig. 5 comprises all the four cases of the wake interaction presented in this study to have a global view of the evolution between the two variables. The distribution of VD with respect to ΔI is in line with the fact that for the case of full-wake interaction (WD207°), ΔI is the largest, which reflects on the data scatter also, as the degree of scatter is high. Then it decreases gradually with the reduction in the degree of wake interaction. As there is less ΔI and scatter in case of the partial-wake interaction (WD220°) than the full interaction case. This is followed by the weak interaction case (WD233°), and finally, the zero interaction case (WD246°) with the lowest ΔI and data scatter.

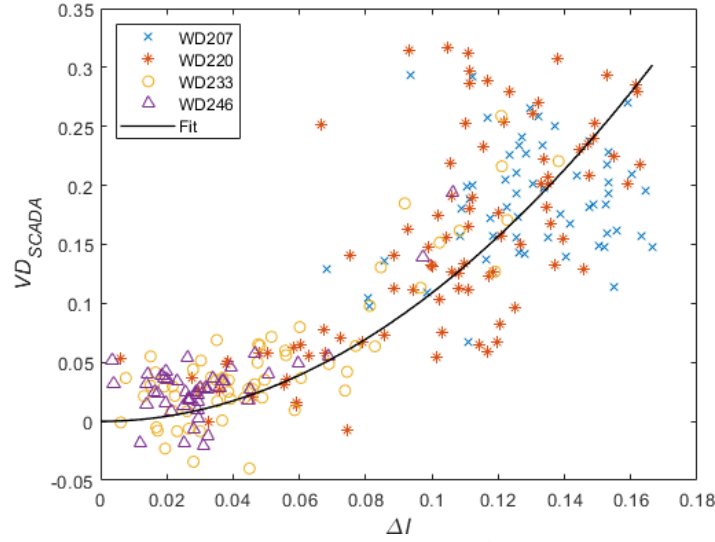


Figure 5: Curve fitting of the evolution of VD_{SCADA} with ΔI at different wake interaction degrees in near-neutral stability conditions

It is noticed that there is a trend between both variables. So, curve fitting was performed to check the best fit for the data as shown in Fig. 5. It was found that the data follows an exponential trend:

$$VD_{SCADA} = a(e^{b\Delta I^2} - 1) \quad (5)$$

where a and b are two fitted coefficients (with 95% confidence bounds); with $a = 526.2$ and $b = 0.0207$. Furthermore, the statistical measures of the goodness of fit are $SSE = 0.8336$, $R - square = 0.6081$, $Adjusted R - square = 0.6066$, and $RMSE = 0.05695$. More details about those statistical measures can be found in [46].

The fit assessment metrics are not the best, especially, the confidence bounds and SSE because of the data scatter.

4.3.2. Wake added turbulence intensity and power deficit

The relationship between the PD, obtained from Eq. (4), and ΔI , calculated using Eq. (6), was investigated (Fig. 6).

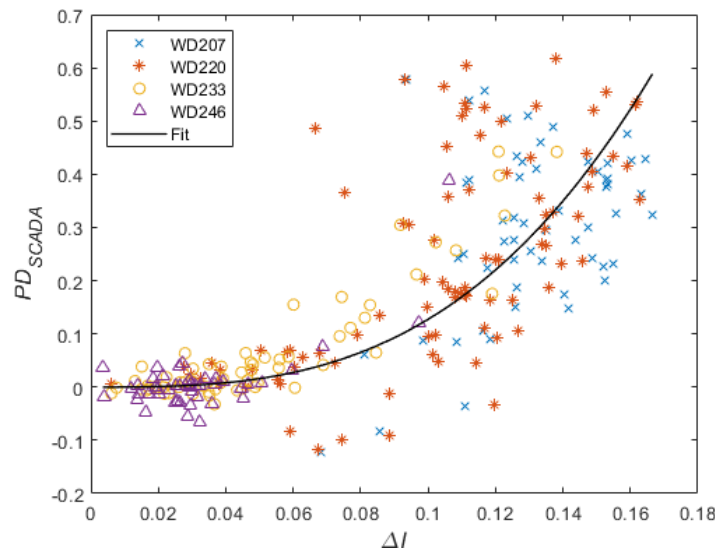


Figure 6: Curve fitting of the evolution of PD_{SCADA} with ΔI at different wake interaction degrees in near-neutral stability conditions

The trend in Fig. 6 looks similar to the one in Fig. 5, which is expected since the generated power is a function of the cube of the WS. Consequently, the same process was held for PD, and a curve fitting was performed. It was found that the data follows an exponential trend like in VD case:

$$PD_{SCADA} = c(e^{d\Delta I^3} - 1) \quad (6)$$

where c and d are two fitted coefficients (with 95% confidence bounds); with $c = 999.4$ and $d = 0.1275$. Moreover, the statistical measures of the goodness of fit are $SSE = 5.147$, $R - square = 0.4885$, $Adjusted R - square = 0.4865$, and $RMSE = 0.1415$.

Like in the case of VD, the fit assessment metrics are not good, which is expected with such scatter.

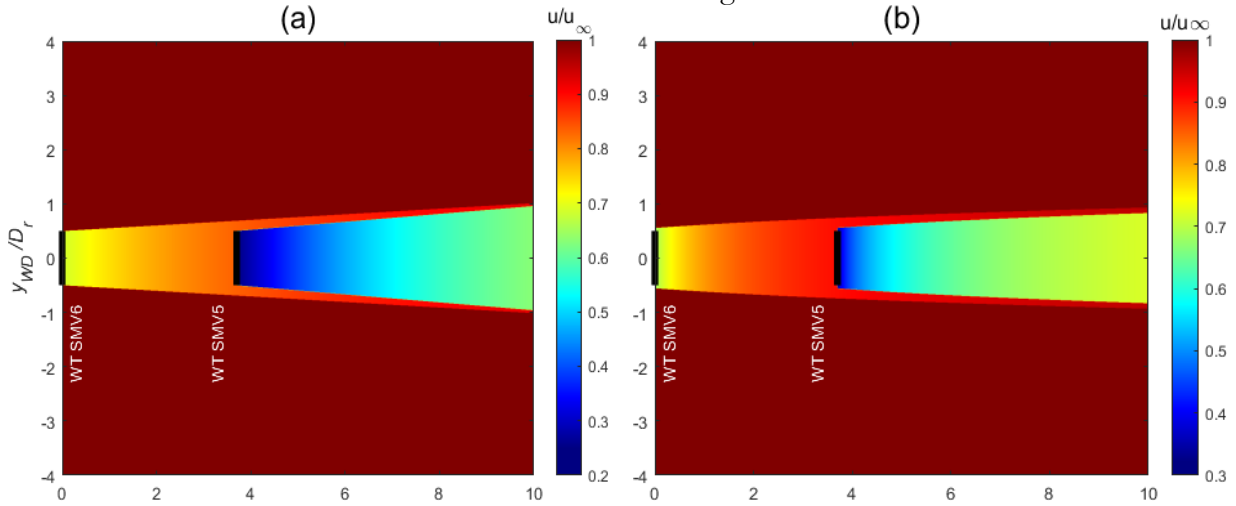
Both laws in Eq. (5) and Eq. (6) are not universal since they will change with other parameters that have not been considered here. Mainly, the relative position of wind turbines, their operating point, and the atmospheric stability conditions. However, it is shown that a clear trend can be extracted, giving confidence in the present handling of categorised SCADA data. Additionally, this feature could be used to better predict the added turbulence within the wakes, or to calibrate the added turbulence intensity models used in combination with wake superposition models. These aspects are out of the scope of the present paper but require further investigation in this direction.

4.4. Comparison of wake models

In this part, a distinct study of the wakes, applying the categorisation process in section 3, is held where a comparison between the different engineering wake models is conducted.

4.4.1. Wake contours

Some wake contours are shown for visualisation purposes to replicate what was done in [1] but using the analytical wake models. For this matter, the normalised velocity contours for the different wake models are shown in the following figures, showing the wake evolution in the horizontal streamwise and crosswise directions at hub height.



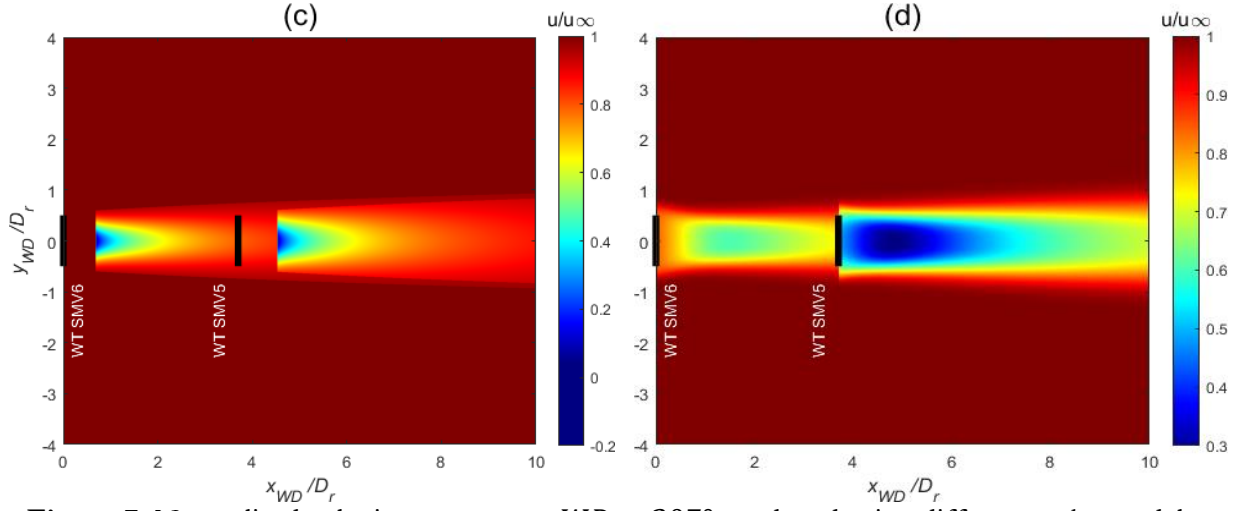


Figure 7: Normalised velocity contours at $WD = 207^\circ$ produced using different wake models, (a) Jensen, (b) Frandsen *et al.*, (c) Bastankhah and Porté-Agel, (d) Blondel and Cathelain.

4.4.2. Velocity deficit from analytical wake models and field measurements

In this part, the VDs obtained from the analytical wake models are compared with the one obtained from the field measurements (LiDAR). This was realised by extracting the velocity at the wake centre assumed at $y = 0$ from the wake contours in Section 4.4.1. The objective is to have a global overview on the performance of the different wake models and superposition methods.

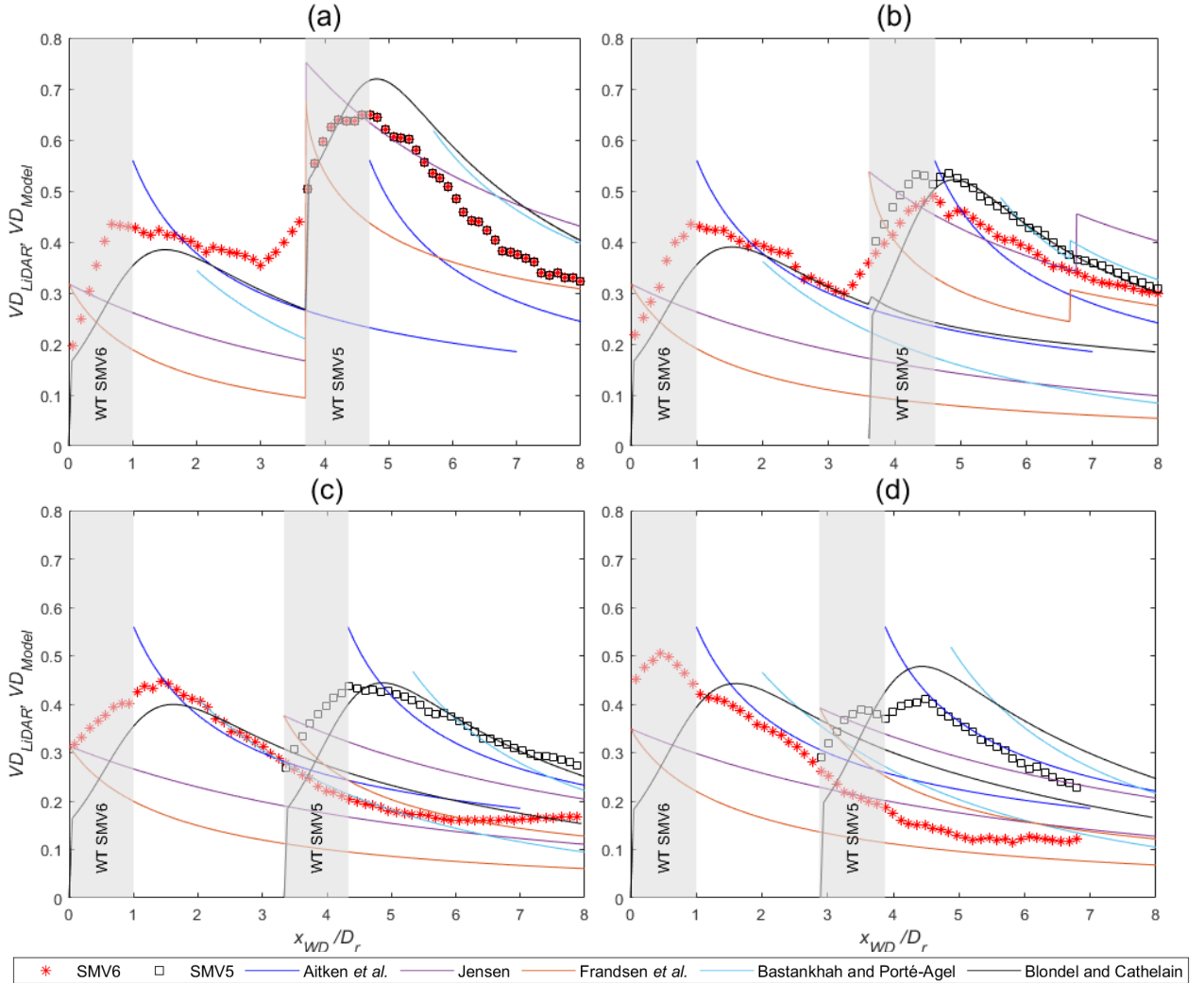


Figure 8: Comparison between the evolution of the normalised velocity deficits (VD) calculated at $y = 0$ and the measurement altitude (z), and obtained from the different engineering wake models superimposed by Lissaman method (VD_{Model}) with the one obtained experimentally (VD_{LiDAR}) in the four WDs depicted from Torres-Garcia *et al.* [1], with respect to the normalised downstream distance (x/D_r). (a) $WD = 207^\circ$, (b) $WD = 220^\circ$, (c) $WD = 233^\circ$, (d) $WD = 246^\circ$.

Fig. 8 shows the comparison between the VD obtained from the different analytical wake models, with respect to the VD presenting the field (LiDAR) measurements, calculated using Eq. (1) for different x/D_r in each of the four WD configurations. The variation in the altitude of each LiDAR data point, (see Appendix D), was considered when applying the wake models to have a reliable comparison [1]. The grey rectangles in the figure mark the area of influence of the WT's where the LiDAR measurements were contaminated by reflection on the rotor [1]. The VD s obtained from the analytical wake models were calculated at the wake centre assumed at $y = 0$ in the horizontal crosswise direction, which is the location of maximal velocity deficit. The VD s extracted from the scanning LiDAR measurements also represent the maximum VD within the wake, but its y -coordinate can be slightly different than 0 [1]. However, it is considered that this does not play an important role in the comparison.

The sudden increase in VD noticed at WT-SMV5 location in Fig. 8a and 8b corresponds to wake interaction with full-wake overlapping for $WD = 207^\circ$ and partial one for $WD = 220^\circ$. Even though there is a small interaction in case of $WD = 233^\circ$ shown in Fig. 8c, there is no obvious increase in the VD like the ones occurring in the overlapping cases, yet there is an interaction which can be detected far downstream as the wake evolves for further distance.

Jensen model neither accounts for the induction zone, nor the near-wake zone. This is not realistic as its VD is always decreasing downstream for a single WT case. In case of a single wake (WT-SMV6), there is always an underestimation of the VD because of the top-hat distribution assumption for wake VD profiles. As it seems to depend on the wake interaction level between WT-SMV6 and WT-SMV5, ranging between 10% - 20% for full interaction ($WD = 207^\circ$). However, this range decreases as the wake interaction level decreases. This is because it is affected by the weakening of the induction zone of WT-SMV5. In case of the wake downstream of WT-SMV5, it depends on the wake interaction level. If there is wake interaction, there is an overestimation, otherwise the underestimation continues like in case of WT-SMV6.

Frandsen model does not consider the induction and near-wake zones like Jensen model, yet nothing was mentioned in the literature about the correct starting downstream position of its application. It also underestimates VD for single wake (WT-SMV6), also because of the top-hat assumption like in case of Jensen, but in this case, the degree of underestimation is even greater than that of Jensen's. Even for the wake of WT-SMV5, underestimation continues, but in this case the wake interaction has the opposite effect on the degree of underestimation, as it is the smallest in case of full interaction, then increases as the interaction level decreases.

Bastankhah and Porté-Agel model is developed to be applicable only in the far-wake. That is why its starting point of application is at a downstream distance of $2D_r$ from the WT according to [13]. Overall, this model seems to perform well, and better than Jensen and Frandsen models. Its overall performance is good either for a single wake (WT-SMV6), or for the superimposed wake (WT-SMV5). It seems that for a single wake, it underestimates the VD with a smaller percentage than that from Jensen and Frandsen *et al.* models, then this percentage decreases as wake interaction decreases. The underestimation continues decreasing as long as there is an interaction of wakes. Once the wake interaction disappears, the model overestimates the VD . On the other hand, for the superimposed wake of WT-SMV5, there is always an overestimation of VD . In some cases of interaction like partial and weak interaction cases, the model performs very well compared to the full and zero interaction cases.

Blondel and Cathelain model accounts for the near-wake in addition to the far-wake zone as shown in Fig. 8. Generally, it performs in a similar manner as Bastankhah and Porté-Agel but produces closer values to the LiDAR measurements. It underestimates the VD for single wake (WT-SMV6) looking at the full-wake interaction, but the underestimation decreases as the wake interaction decreases, then overestimation occurs when interaction vanishes. For the superimposed wake, the model does very well in case of partial and weak interaction compared to the full and zero interaction cases where there is an overestimation. Overall, the model performs significantly well in the cases of partial and weak-wake interaction compared to the other two cases.

Additionally, Aitken *et al.* model performs very well generally in the single wake cases (WT-SMV6), and in the superimposed wake (WT-SMV5) in some configurations.

4.4.3. Performance assessment of analytical wake models

The performances of the analytical wake models and the wake superposition methods are investigated by applying the performance metrics to find out which model returns the closest results to the field measurements.

Since the purpose of the analytical wake models is to describe the wake of a single wind turbine, the performance metrics for the zero-wake interaction case were firstly computed, to specify which wake model works the best when compared to the field measurements. The case of zero-wake interaction comprises WT-SMV6 in the four WD configurations, in addition to WT-SMV5 for the $WD = 246^\circ$ configuration as shown in Table 4. Afterwards, the metrics for the cases with wake interaction were calculated to specify the wake superposition method that works best with the specified wake model as shown in Table 5.

During calculating the performance metrics for the different wake models for WT-SMV6, the points lying in the induction zone upstream of WT-SMV5 (which starts with the sudden increase in VD of the single wake (WT-SMV6)) were disregarded. So, they should not be considered while checking the figures. However, for superimposed wake (WT-SMV5), the performance of the different superposition methods becomes important.

Table 4: Performance metrics for the best wake model specification. The model with the optimum values is shown together with its absolute values of the maximum and minimum deviations from the ideal value of each metric.

Case	SMV6 (4WDs) & SMV5 (WD246°)
Interaction level	Zero interaction
FB ($-2 < FB < 2$)	Blondel and Cathelain (0.03 - 0.23)
MG ($0.75 < MG < 1.25$)	Blondel and Cathelain (0.79 - 1.11)
NMSE ($NMSE \leq 0.5$)	Blondel and Cathelain (0.005 - 0.05)
VG ($0.75 < VG < 1.25$)	Blondel and Cathelain (1.004 - 1.06)
R (Ideal = 1)	Jensen (0.9 - 0.93)
Mean FAC2 (Ideal =1)	Blondel and Cathelain (0.9 - 1.3)

Table 4 summarises the performance metrics calculated for all the wake models used in the study. For each metric, the wake model with the best results was chosen. According to Table 4, Blondel and Cathelain model dominates significantly in almost all the metrics. It returns the closest result to the ideal value of each metric. Therefore, Blondel and Cathelain model is considered as the best performing wake model. Consequently, the wake superposition method which works best with Blondel and Cathelain model is specified afterwards in Table 5.

Table 5: Performance measures for the best wake superposition method specification with Blondel and Cathelain wake model. The method with the optimum value in each case is shown together with the value of the metric.

Case	SMV5 (WD207°)	SMV5 (WD220°)	SMV5 (WD233°)
Interaction level	Full interaction	Partial interaction	Weak interaction
FB ($-2 < FB < 2$)	Katić <i>et al.</i> (0.14)	Lissaman (0.01)	Lissaman (0.001)
MG ($0.75 < MG < 1.25$)	Lissaman (0.82)	Lissaman (1.02)	Lissaman (1.003)
NMSE ($NMSE \leq 0.5$)	Lissaman (0.03)	Lissaman (0.001)	Lissaman (0.002)
VG ($0.75 < VG < 1.25$)	Lissaman (1.042)	Lissaman (1.002)	Lissaman (1.002)
R (Ideal = 1)	Lissaman (0.96)	Lissaman (0.96)	Katić <i>et al.</i> (0.96)
Mean FAC2 (Ideal = 1)	Katić <i>et al.</i> (0.91)	Lissaman (0.982)	Lissaman (0.997)

Therefore, according to Tables 4 and 5, Blondel and Cathelain model coupled with Lissaman wake superposition method was concluded to perform the best. The same procedure used to generate Table 5, was used for Jensen, Frandsen *et al.* and Bastankhah and Porté-Agel models to have the comparison figures with each wake model coupled with the wake superposition method that works best with it. It was found that Lissaman method works best with Jensen, Frandsen *et al.* and Bastankhah and Porté-Agel models too.

5. Conclusions and future work

A scanning LiDAR measurement campaign was conducted for a 7-month duration in SMARTEOLE project to study the wake behaviour and characteristics. The campaign was set up such that the wakes of two WT's were measured. Four WDs were selected (207°, 220°, 233° and 246°) corresponding to different wake interaction levels. Only near-neutral atmospheric boundary layer was considered in the wake models comparison study.

MERRA-2 dataset was used for the categorisation of data, and the post-processing of the scanning LiDAR measurements was previously done by Torres-Garcia *et al.* [1].

The aim of the present study was to validate the engineering wake models in comparison with full scale WT's wake evolution obtained with scanning LiDAR measurements. This required complementing the LiDAR measurements with WT's SCADA data. The main objective was to obtain the operational and environmental data acquired by SCADA during the LiDAR measurement campaign to feed the analytical wake models to mitigate the uncertainties on specific inputs as the turbulence intensity and the WT thrust coefficient. However, another secondary objective was to find or validate relationships between the specific data such as the atmospheric stability and the free-stream turbulence intensity, and velocity and power deficits with respect to the wake added turbulence intensity.

The correlation between the atmospheric thermal stability and the free-stream turbulence intensity (I_u) was investigated. Different WSs were involved to consider different wind strengths, and, thus, different atmospheric stability conditions: neutral, stable, and convective atmospheric boundary layers, expressed with Monin-Obukhov Length (L). As expected, a smaller scatter was noticed in I_u in case of high winds, and the atmospheric stability conditions tended towards the neutral stability configuration. However, in case of light winds, there was much more scatter in I_u , and the atmospheric stability conditions tended towards convective or stable configurations.

While investigating the relationship between the wake added turbulence intensity (ΔI), a clear trend was detected between the velocity and power deficits with respect to ΔI . Consequently, empirical equations relating those variables were obtained by curve fitting of the data. It was found that both velocity and power deficits follow an exponential trend with respect to ΔI .

Before applying the engineering wake models coupled with the wake superposition methods, a performance assessment of the different wake models was held first by calculating different statistical error metrics in case of zero-wake interaction, which showed how close the wake models predictions compared to the field LiDAR measurements. Blondel and Cathelain wake model was the one with the best results compared to the rest of the models. Afterwards, another performance assessment of the wake superposition methods in case of wake interaction, when coupled with Blondel and Cathelain model, was conducted. Based on the assessment results,

Lissaman superposition method was found to perform the best with Blondel and Cathelain model.

Regarding future work, the usage of the more sophisticated ERA5 reanalysis would be better suited, as it has proven its superiority [47]. Indeed, the ERA5 reanalysis is based on a refined grid with higher spatial and temporal resolutions. Furthermore, the data is at 100 m, which is closer to the hub height than the data from MERRA-2 at 50 m. Also, it is important to study other atmospheric stability conditions as in Aubrun *et al.* [28] since atmospheric stability is not properly considered neither in experiments nor in analytical wake models [5].

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- Appendix A: Engineering wake models

- **Aitken *et al.* model:**

An empirical wake model that assumes a power-law relationship of the VD with downstream distance x_{WD} . The model was developed based on the fitting of a collection of measurements dated between 1981 and 2013 [1,25], and defined as:

$$VD(x_{WD}) = 0.56 \left(\frac{x_{WD}}{D_r} \right)^{-0.57} \quad (A.1)$$

- **Jensen model:**

Jensen model is derived by applying the conservation of mass to a control volume downwind of the wind turbine, and then using Betz theory to relate the wind speed just behind the rotor to the turbine thrust coefficient C_T [5,38]. It neglects the near-wake region, and assumes a top-hat distribution, for the velocity deficit in the wake for the sake of simplicity [5,9]. The normalized velocity deficit based on this model is given by:

$$\frac{\Delta u}{u_o} = \frac{1 - \sqrt{1 - C_T}}{(1 + 2k\tilde{x})^2} \quad (A.1)$$

where u_o is the mean incoming wind speed, and obtained from SCADA for WT-SMV6, and $\Delta u = u_o - u_w$, where u_w is the wind speed within the wake region, and k is the wake growth rate, which is obtained from $k = 0.4I_u$, with I_u , provided in Table 2, corresponding to each WT in a certain WD, is utilised. While the wake diameter, $D_w(x) = D_r + 2kx$, is assumed to expand linearly as a function of the downstream distance at a rate k .

- **Frandsen *et al.* model:**

Frandsen *et al.* [12] applied both mass and momentum conservation equations over a control volume surrounding the turbine to derive their model. Similar to Jensen model, the top-hat shape was assumed for velocity-deficit profiles in the wake [5,12,13]. The model is defined as follows:

$$\frac{\Delta u}{u_o} = 0.5 \left(1 - \sqrt{1 - \frac{2C_T}{\beta + \alpha\tilde{x}}} \right) \quad (A.2)$$

where α is the wake expansion factor, equal to $10k$, while β is function of C_T , and it is meaningful for values of C_T between 0 and 1 [5,12], which was ensured before using the model. Additionally, $D_w(x) = D_r(\beta^{1.5} + \alpha\tilde{x})^{\frac{1}{3}}$, and β is expressed by:

$$\beta = \frac{1 + \sqrt{1 - C_T}}{\sqrt{1 - C_T}} \quad (A.3)$$

- **Bastankhah and Porté-Agel model:**

Bastankhah and Porté-Agel [13] developed an extension of Jensen model in which the linear wake expansion is retained, but the overly simplified top-hat profile is replaced with a Gaussian self-similar profile, which more closely approximates a classic far-wake profile [5,13]. It is described by:

$$\frac{\Delta u}{u_o} = \left(1 - \sqrt{1 - \frac{C_T}{8(\sigma/D_r)^2}} \right) \exp \left(-0.5 \left[\frac{y^2}{\sigma^2} + \frac{(z - z_h)^2}{\sigma^2} \right] \right)$$

with $\sigma = k_w x + 0.2\sqrt{\beta}D_r$ the standard deviation of the Gaussian-like velocity deficit profiles at each x as stated in [13], and k_w the wake growth rate for this model taken as $k_w \approx 0.3837I_u + 0.0037$ [5,32,40]. I_u is provided in Table 2.

- **Blondel and Cathelain model:**

Blondel and Cathelain model [15] takes into account the near-wake zone, as the absolute Gaussian shape is replaced with a super-Gaussian shape, in addition to preserving the mass and momentum conservation. The super-Gaussian function tends towards a top-hat shape for high values of the super-Gaussian order n in the near-wake conditions, and n decreases gradually with the downstream distance x . This gradual decrease involves a gradual transition in the shape of the wake velocity profiles, from super-Gaussian to Gaussian. Once $n = 2$, the traditional Gaussian shape is recovered, which signifies reaching the far-wake region. It is expressed by:

$$\frac{\Delta u}{u_o} = \left(2^{\frac{2}{n}-1} - \sqrt{2^{\frac{4}{n}-2} - \frac{nC_T}{16\Gamma(2/n)\tilde{\sigma}^{4/n}}} \right) \exp \left(-\frac{\tilde{r}^n}{2\tilde{\sigma}^2} \right) \quad (\text{A.5})$$

where Γ is the Gamma function, while $\tilde{\sigma}$ and \tilde{r} are the standard deviation and the radial distance from the wake centre respectively. The tilde symbol denotes a normalisation by the wind turbine diameter, D_r . The wake characteristic width is $\tilde{\sigma} = (0.17I_u + 0.005)\tilde{x} + 0.2\sqrt{\beta}$ in the original version of the model in [15], but after the further calibration and validation performed in [37], it became $\tilde{\sigma} = (0.18I_u + 0.0119)\tilde{x} + (0.0564C_T + 0.13)\sqrt{\beta}$. Regarding n , it can be obtained using; $n \approx a_f e^{b_f \tilde{x}} + c_f$, with $a_f = -3.11$, $b_f = -0.68$, and $c_f = 2.41$ according to the original model [15], however, after calibration in [37], a_f takes the value which ensures that the maximum normalised velocity deficit at the disk, $VD_{max}(0)$, equals the axial induction factor ($a = 0.5[1 - \sqrt{1 - C_T}]$), $b_f = 1.59e^{-23.31I_u} - 2.15$, and $c_f = 2.98$. This development in the model is explained in both Figs. A.1 & A.2. It fair to say that n differs from one wind farm to another depending on the operating conditions.

Fig. A.1 shows the evolution of n with respect to x/D_r for both versions. It appears that n does not reach 2. Hence, the typical Gaussian shape of the far-wake is not imposed even at $x/D_r = 8$, which is confirmed in Fig. A.2. This is only correct for the old model. For the new model, n does not reach 2 either. However, it produces considerably improved results, and the Gaussian shape is imposed in the far-wake as shown in Fig. A.2. This demonstrates the improvement incorporated in the new model.

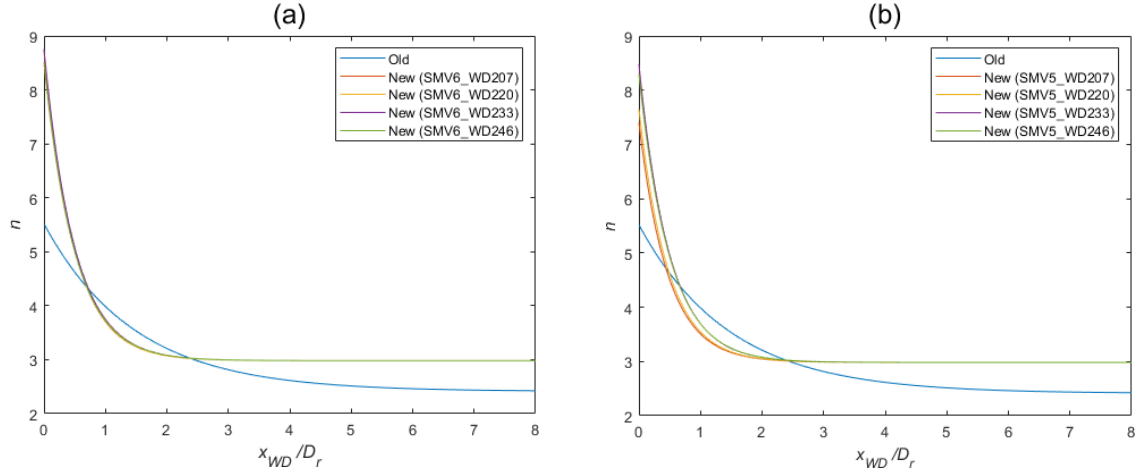


Figure A.1: Evolution of the super-Gaussian order (n) with the normalised downstream distance (x/D_r) for both the old and new versions of Blondel and Cathelain wake model, with (a) WT-SMV6, (b) WT-SMV5

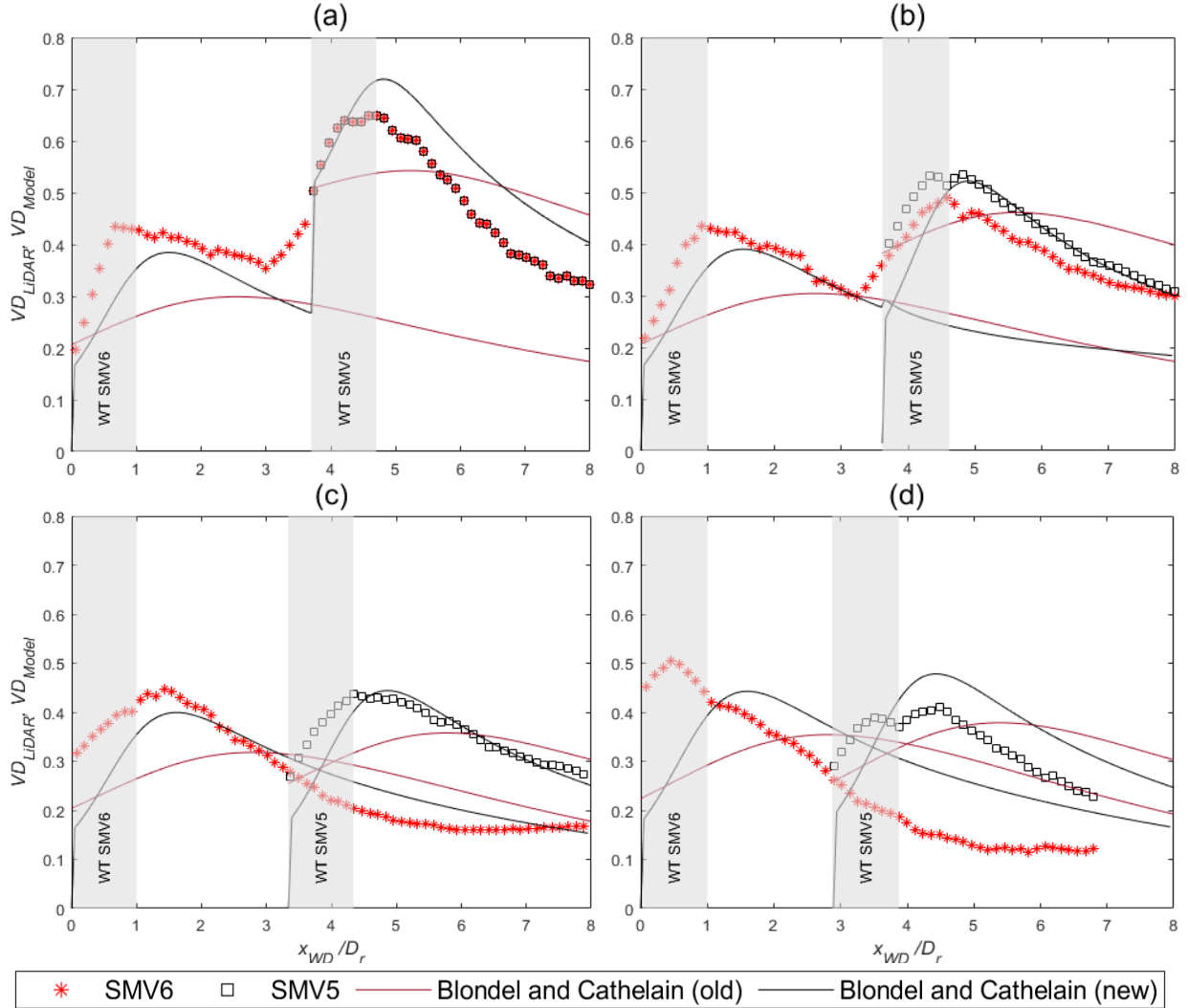


Figure A.2: Comparison between the evolution of the normalised velocity deficits (VD) calculated at $y = 0$ and the measurement altitude (z), and obtained from both the old and new versions of Blondel and Cathelain wake model superimposed by Lissaman method (VD_{Model}), with the one obtained experimentally (VD_{LIDAR}) in the four WDs depicted from Torres-Garcia *et al*

al. [1], with respect to the normalised downstream distance (x/D_r). (a) $WD = 207^\circ$, (b) $WD = 220^\circ$, (c) $WD = 233^\circ$, (d) $WD = 246^\circ$.

- Appendix B: Wake superposition methods

- **Lissaman method:**

Lissaman's technique is based on the linear superposition of velocity deficit, and is defined as in [36]:

$$u(X) = u_\infty - \sum_{i=1}^n \Delta u_i(X) \quad (\text{B.1})$$

where $\Delta u_i(X) = u_\infty - u_i(X)$.

- **Katić *et al.* method:**

This technique is based on the linear superposition of energy deficit, and is expressed as in [38]:

$$u(X) = u_\infty - \sqrt{\sum_{i=1}^n \Delta u_i^2(X)} \quad (\text{B.2})$$

where $\Delta u_i(X) = u_\infty - u_i(X)$.

- **Voutsinas *et al.* method:**

Similar to Katić *et al.* method, this technique is based on the linear superposition of energy deficit, and is given as in [39]:

$$u(X) = u_\infty - \sqrt{\sum_{i=1}^n \Delta u_i^2(X)} \quad (\text{B.3})$$

where $\Delta u_i(X) = u_{in,i} - u_i(X)$.

- **Niayifar and Porté-Agel method:**

Similar to Lissaman's method, this method is based on the linear superposition of velocity deficit, and is given as in [40]:

$$u(X) = u_\infty - \sum_{i=1}^n \Delta u_i(X) \quad (\text{B.4})$$

where $\Delta u_i(X) = u_{in,i} - u_i(X)$.

Although, Lissaman, and Niayifar and Porté-Agel share the same approach, and the same case applies for Katić *et al.*, and Voutsinas *et al.*, however, there is a difference when it comes to calculating $\Delta u_i(X)$. As it is calculated based on the undisturbed incoming boundary-layer flow speed, u_∞ , in case of Lissaman and Katić *et al.*, however, it is calculated using, $u_{in,i}$ the incoming flow speed for that (i th) wind turbine [5] in case of Voutsinas *et al.* and Niayifar and Porté-Agel. So, the differences in the superposition methods arise from; 1) Different superposition principles; linear superposition of velocity deficit or energy deficit, 2) Different definitions of the velocity deficit caused by the i th turbine.

- Appendix C: Performance measures

There are two types of errors: systematic error (e.g., uncalibrated sensor), which refers to the ratio of the model predictions (C_p) to the observations (C_o), and random error, which is due to unpredictable fluctuations [44]. Each one of the measures indicates one of them, or sometimes both. Those performance measures are given as follows [44,45]:

- **Fractional Bias (FB):** Measures only the mean relative bias indicating the systematic error of a model that refers to the arithmetic difference between C_p and C_o , and is based on a linear scale. FB is expressed by:

$$FB = \frac{(\overline{C_o} - \overline{C_p})}{0.5(\overline{C_o} + \overline{C_p})} \quad (C.1)$$

- **Geometric Mean Bias (MG):** Does the same task as FB, however, the difference between them is that MG is based on a logarithmic scale, which is more suitable in case C_p and C_o vary by many orders of magnitude, and it is described by:

$$MG = e^{\overline{\ln(C_o)} - \overline{\ln(C_p)}} \quad (C.2)$$

- **Normalised Mean Squared Error (NMSE):** A measure of the mean relative scatter and describes both systematic and random errors. It is based on a linear scale like FB, and is given by:

$$NMSE = \frac{\overline{(C_o - C_p)^2}}{\overline{C_o} \overline{C_p}} \quad (C.3)$$

- **Geometric Variance (VG):** A measure of the mean relative scatter as well as NMSE, but based on a logarithmic scale, and is given by:

$$VG = e^{\overline{(\ln(C_o) - \ln(C_p))^2}} \quad (C.4)$$

- **Correlation Coefficient (R):** Demonstrates the linear relationship between two variables, and expressed by:

$$R = \frac{\overline{(C_o - \overline{C_o})(C_p - \overline{C_p})}}{\sigma_{C_o} \sigma_{C_p}} \quad (C.5)$$

- **The fraction of predictions within a factor of two of observations (FAC2):** The most robust measure, because it is not overly influenced by high and low outlier, and it is described by:

$$0.5 \leq FAC2 = \frac{C_p}{C_o} \leq 2 \quad (C.6)$$

where the overbar \overline{C} denotes the average value all over the dataset, and σ_C expresses the standard deviation all over the dataset.

- Appendix D: LiDAR measurements altitudes

Z_{SMV6}				Z_{SMV5}			
207°	220°	233°	246°	207°	220°	233°	246°
99.11	99.14	99.06	99.01	86.04	85.69	85.71	85.86
98.63	98.58	98.44	98.37	85.63	85.19	85.12	85.23
98.14	98.02	97.82	97.72	85.22	84.68	84.54	84.60
97.67	97.47	97.20	97.08	84.82	84.18	83.96	83.96
97.19	96.91	96.58	96.43	84.41	83.69	83.38	83.33
96.71	96.36	95.97	95.79	84.01	83.20	82.79	82.70
96.24	95.80	95.35	95.14	83.62	82.71	82.21	82.07
95.77	95.25	94.73	94.50	83.23	82.22	81.63	81.44
95.29	94.70	94.12	93.86	82.84	81.73	81.06	80.81
94.83	94.15	93.50	93.21	82.45	81.25	80.48	80.18

94.36	93.59	92.89	92.57	82.06	80.76	79.91	79.55
93.90	93.05	92.27	91.92	81.68	80.28	79.34	78.92
93.44	92.50	91.65	91.28	81.31	79.81	78.77	78.29
92.98	91.96	91.04	90.63	80.93	79.34	78.19	77.66
92.52	91.42	90.42	89.99	80.56	78.87	77.62	77.03
92.07	90.88	89.81	89.34	80.20	78.39	77.05	76.40
91.61	90.33	89.19	88.70	79.84	77.92	76.48	75.78
91.16	89.79	88.58	88.05	79.48	77.45	75.91	75.15
90.72	89.25	87.97	87.41	79.12	76.99	75.35	74.52
90.27	88.71	87.36	86.76	78.77	76.54	74.78	73.90
89.83	88.18	86.75	86.12	78.42	76.08	74.22	73.27
89.39	87.65	86.14	85.47	78.08	75.62	73.66	72.64
88.95	87.12	85.53	84.83	77.74	75.17	73.10	72.02
88.51	86.59	84.92	84.19	77.40	74.72	72.54	71.39
88.08	86.06	84.31	83.54	77.07	74.28	71.98	70.77
87.65	85.53	83.70	82.90	76.74	73.84	71.42	70.14
87.23	85.00	83.09	82.25	76.42	73.40	70.87	69.52
86.80	84.47	82.48	81.61	76.10	72.96	70.31	68.89
86.38	83.95	81.88	80.96	75.79	72.52	69.76	68.27
85.96	83.43	81.27	80.32	75.47	72.10	69.21	67.65
85.55	82.91	80.66	79.67	75.17	71.67	68.66	67.02
85.13	82.40	80.06	79.03	74.86	71.25	68.12	66.40
84.72	81.88	79.45	78.39	74.56	70.83	67.57	65.78
84.32	81.37	78.85	77.74	74.27	70.41	67.03	—
83.91	80.85	78.24	77.10	73.98	70.00	66.49	—
83.51	80.34	77.64	76.45	73.70	69.59	65.96	—
83.11	79.83	77.04	75.81	73.42	69.18	65.42	—
82.71	79.33	76.44	75.16	73.07	68.78	64.88	—
82.32	78.82	75.84	74.52	—	68.38	64.35	—
81.93	78.32	75.24	73.88	—	67.99	63.82	—
81.54	77.82	74.64	73.23	—	67.54	—	—
81.16	77.32	74.04	72.59	—	67.02	—	—
80.78	76.82	73.44	71.94				
80.41	76.33	72.85	71.30				
80.04	75.84	72.25	70.65				
79.67	75.35	71.66	70.01				
79.30	74.87	71.06	69.37				
78.94	74.38	70.47	68.72				
78.58	73.89	69.88	68.08				
78.23	73.42	69.29	67.44				
77.88	72.94	68.70	66.79				
77.53	72.47	68.11	66.15				
77.19	72.00	67.52	65.50				
76.85	71.53	66.93	64.86				
76.52	71.06	66.35	64.22				
76.19	70.60	65.76	63.57				
75.86	70.14	65.18	—				
75.54	69.68	64.60	—				
75.22	69.23	64.02	—				

74.91	68.77	63.44	—
74.60	68.33	62.86	—
74.29	67.89	62.28	—
73.99	67.45	61.71	—
73.69	67.01	61.13	—
73.40	66.57	60.56	—
73.12	66.14	59.99	—
72.83	65.72	59.42	—
72.56	65.29	—	—
—	64.87	—	—
—	64.45	—	—
—	64.04	—	—
—	63.64	—	—
—	63.23	—	—
—	62.83	—	—

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